

Literature Review on Multi-Regional Input-Output Matrices (EE-MRIO)

Rodrigo Aylmer

COPPE Universidade Federal do Rio de Janeiro

Dr. Roberto Aylmer

Rennes School of Business, France

Dr. Mariana Aylmer

Rennes School of Business, France

Dr. Murillo Dias

Fundação Getulio Vargas, Brazil

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ABSTRACT: *The aim of this paper is to review the development of environmentally extended multi-regional input-output matrices (EE-MRIO) through a literature review. The EE-MRIO, developed by Wassily Leontief, reveals the interdependence of industries in an economy, using general-equilibrium models to express relationships between sectors. This article enhances the current knowledge on the subject, providing valuable insights for scholars, decision-makers, and practitioners, and offers a comprehensive overview of the models' evolution.*

KEYWORDS: Multi-Regional Input-Output Matrices; Biofuel; Bio methanol; Industry

INTRODUCTION

This article addresses the analysis of the EE-MRIO, from its conception to its extension to multi-regional input-output models, following Cabernard *et al.* (2019). An economy works by equating the balance between supply and demand within a vast network of related activities (Guilhoto, 2011). In production processes, certain elements are created (*outputs*) from other elements (*inputs*) and are, in turn, used and consumed in other production processes, so there are multiple simultaneous causal relationships (Leontief, 1991).

Firstly, Wassily Leontief (1906-1999) developed a methodology capable of revealing the interdependence of the industries of the economy, namely, which sectors are acquiring products and which sectors are supplying these products. The input-output (I-O) model is built based on observed economic data and shows the interrelationship between industries or sectors that produce outputs and consume inputs in their production process. (Owen, 2015; Miller & Blair, 2009). I-O models are also called "general-equilibrium models" and, therefore, imply that in an economy, the total inputs equal the total

outputs. (Owen, 2015; Leontief, 1936) In its simplest form, the relationships between the sectors of the I-O model are expressed through two-dimensional intersectoral transaction tables, in which each row and each column represent a different sector of the industry that sells and buys products, respectively (Guilhoto, 2011; Miller & Blair, 2009).

In I-O-based models, the sectors' products can be consumed as intermediate inputs for the production processes of other sectors. The final demand can also consume them directly¹, as shown in Figure 1. The sum of final demand and intermediate consumption provides the total output of the sector that the economy demands (Guilhoto, 2011).

It is important to note that, as industries are buying and selling with each other, it is possible to record data both in physical units (e.g., ton of steel, m³ of water, among others) and in monetary units (e.g., dollars). However, as Miller & Blair (2009) point out, as sectors generally trade more than one product², the representation of the equivalent in price of the physical good traded is more convenient than the physical representation.

During the production process, taxes are levied, products are imported and enter the production process, added value³ is generated and jobs are created (Guilhoto, 2011). The model assumes that only domestic products can be directly exported, and that, therefore, imported products must go through some production process before being exported, as illustrated in Figure 1:

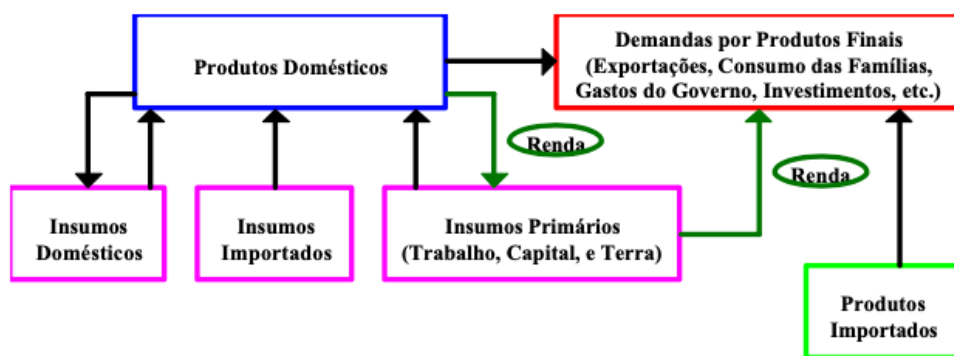


Figure 1 Flowchart of the Input-Output model. Source: Guilhoto (2011)

¹ According to Miller and Blair (2009), the final demand is not a simple concept. It's the sales of each sector to the end markets that consume its products. In its simplest form, it is composed of a column vector, but it is possible to include different components for the final demand, such as: Export of goods and services, government consumption, residential consumption, investments, among others. This complexity and versatility is what makes the study of final demand so intriguing.

² Take, for example, an automobile sector that produces two distinct models of automobiles, A and B. A physical representation would have to consider the units sold from A and B to each purchasing sector of the economy. A monetary representation would simplify the process, as it can provide the value that the automobile industry received from the sales of the two models in a single datum (Miller and Blair, 2009).

³ Value added is the profit, or economic benefit, received by the sectors annually. (KITZES, 2013). Miller and Blair (2009) define added value as the primary *inputs* in addition to intersectoral exchanges, which are necessary for the production process.

Figure 1 shows that the economy's⁴ income is generated through the remuneration of labor, capital, and agricultural land. The government is remunerated through taxes levied on individuals and businesses. The final demand for imports directly consumes or uses them as inputs to produce domestic goods. The relations between the sectors are expected to be non-homogeneous since the relationship depends on the *inputs that* each sector exchanges with each other, and these *inputs* depend on the non-homogeneous production processes. Thus, the intensity of these relationships is the main point of Leontief's input-output analysis (Guilhoto, 2011).

METHODOLOGY

This research study is an exploratory approach that uses qualitative, inductive, interpretive, constructionist, idiographic, and cross-sectional methods. The study involves conducting a comprehensive literature review on the topic.

BASIC THEORY

Let Z be a matrix such that each element z_{ij} represents the demands of sector j over the course of a year for the goods and services provided by sector i . The element z_{ij} can be understood as the money flows received by the i th supplying sector for *the inputs* directed to the j th producing sector of the economy (Leontief, 1936). The Z matrix represents the intersectoral demand of the economy (Miller & Blair, 2009).

Let X_i and Y_i be column-vectors in which each element x_i and y_i represent the total output of each sector and the final demand of the economy for the products of the i th sector, respectively. Assuming that the economy is made up of n sectors, the total output of the i th sector can be expressed as the sum of the multiple intersectoral demands z_{ij} with the final demand:

$$x_i = z_{i1} + z_{i2} + z_{i3} + \dots + z_{in} + y_i = \sum_{j=1}^n z_{ij} + y_i \quad (1)$$

Assuming that the intermediate flows per unit of output are fixed (Guilhoto, 2011), one can define the matrix A , such that each element a_{ij} is equal to the intersectoral exchange z_{ij} divided by the total output of sector j , i.e., the monetary value of the resources that sector j demands from sector i to produce a unit of value, or the demanded rate of *inputs* from sector j for the products from sector i (Miller & Blair, 2009). In other words, if the input-output matrix is working with the dollar, the element a_{ij} represents the dollar value that sector j demands from sector i to generate one dollar of total production⁵ (Kitzes, 2013).

$$A \rightarrow a_{ij} = \frac{z_{ij}}{x_j} \quad (2)$$

⁴ Irving Fisher (1906, p. 52) defines income as "a flow of services over a definite period of time" (our translation), while capital is a "photograph of wealth" existing at a given instant (ibid., p. 66).

⁵ It is important to note that the element a_{ij} by essence is a coefficient between 0 and 1, such that the sum of the columns of a_{ij} = 1. They are, therefore, coefficients of proportion of the relevance of the supplying sector i in the total production of the purchasing sector j . This definition will be important for the definition of the L matrix, as will be seen later.

Matrix A is also called the matrix of direct input coefficients, or matrix of technical coefficients⁶ (Guilhoto, 2011; Miller & Blair, 2009). You can rewrite equation (1) by entering equation (2):

$$x_i = a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \dots + a_{ij}x_j + y_i$$

Which can be written in the generalized way:

$$\sum_{j=1}^n a_{ij}x_j + y_i = x_i \quad (3)$$

$$i = 1, 2, \dots, n$$

If rewritten in matrix form, we have:

$$AX + Y = X \quad (4)$$

Where A is a square matrix of order (nxn), X and Y are vectors columns of order (nx1). Solving the matrix system in x, we obtain:

$$Y = X - AX \quad (5)$$

$$Y = (I - A).X \quad (6)$$

$$X = (I - A)^{-1}.Y$$

$$X = L.Y \rightarrow L = (I - A)^{-1}$$

Thus, it is possible to define the total production X as a function of the final demand Y. The matrix L is also called the Leontief matrix, or matrix of direct and indirect coefficients. According to Guilhoto (2004), each l_{ij} element of the L matrix should be interpreted as the total production of sector i that is necessary to produce a unit of final demand in sector j.

Matrix equation (6) can be opened as follows:

$$\begin{aligned} x_1 &= l_{11}y_1 + l_{12}y_2 + l_{13}y_3 + \dots + l_{1n}y_n \\ x_2 &= l_{21}y_1 + l_{22}y_2 + l_{23}y_3 + \dots + l_{2n}y_n \\ &\vdots \\ x_n &= l_{n1}y_1 + l_{n2}y_2 + l_{n3}y_3 + \dots + l_{nn}y_n \end{aligned} \quad (7)$$

Equation (7) shows that total production is intrinsically related to final demand, so that by increasing the final demand of a sector y_n , it is possible to determine how the total production of each industry changes following the proportion of the corresponding term of the L matrix (Owen, 2015).

Moore & Petersen (1955, p.380) go further, saying that "the inverse matrix [L] shows the direct and indirect requirements of each industry, associated with a given final demand, after all the requirements have played their part through the system"⁷

⁶ The technical coefficients establish fixed proportions of the relationships between the *inputs* and *outputs* of the sectors. The consequence of choosing this form of representation is that economies of scale in production are ignored (Miller & Blair, 2009), which does not occur in reality, and is a simplification made by the model.

⁷ "The inverse matrix shows the direct and indirect requirements from each industry associated with a given final demand after all requirements have had a chance to work their way through the system."

Thus, the interconnected structure of the economy works as follows: by increasing the final demand of one sector j , the total output of the other supplying sectors of sector j is also increased⁸. However, in order to carry out their productive effort, the other sectors, which supply inputs to sector j , need their own inputs from other sectors, which, in turn, need their own inputs. At some point, the sector itself is a supplier of inputs to other sectors, which makes the economy rotate in a cyclical multiplicative process (Guilhoto, 2011). Figure 2 resumes the process in a conceptual way.

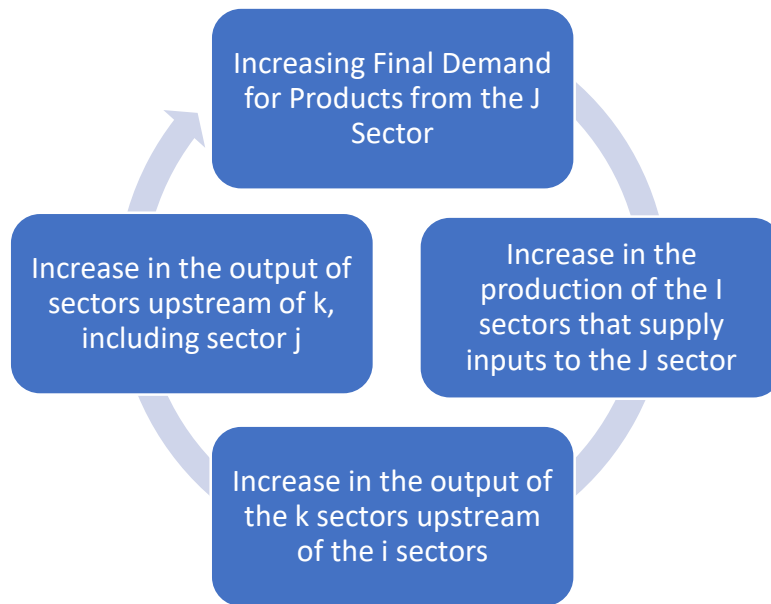


Figure 2 Conceptual scheme of the multiplicative process of demand in the economy.

One way to mathematically explain the multiplicative process of the final demand and consequently the economic meaning of the model proposed by Leontief is through Taylor's expansion (Owen, 2015, Miller & Blair, 2009, Bjerkholt & Kurz, 2006, Guilhoto, 2011). Whereas:

$$L = (I - A)^{-1} = I + A + A^2 + A^3 + \dots + A^n \quad (8)$$

It is possible to prove the equality of equation (8) by multiplying both sides of the equation by $(I - A)$:

$$\begin{aligned} (I - A) \cdot (I - A)^{-1} &= (I - A) \cdot (I + A + A^2 + A^3 + \dots + A^n) \\ I &= (I - A) \cdot I + (I - A) \cdot A + (I - A) \cdot A^2 + (I - A) \cdot A^3 + \dots + (I - A) \cdot A^n \\ I &= I - A + A - A^2 + A^2 - A^3 + A^3 + \dots + A^n - A^n - A^{n+1} \\ I &= I - A^{n+1} \end{aligned}$$

⁸ In other words, in order to increase one unit of the final demand y_i , the total output of sector i must grow in l_{ij} , since it is a supplier of inputs to sector j .

Since all a_{ij} elements of matrix A have values between 0 and 1, making n tend to infinity⁹, it is possible to consider the term A^{n+1} to be zero. Thus, we can start from equations (6) and (8) to identify what happens to the total production when we vary the final demand as follows:

$$\begin{aligned} X &= L \cdot Y \\ \Delta X &= L \cdot \Delta Y \\ L &= (I - A)^{-1} = I + A + A^2 + A^3 + \dots + A^n \\ \Delta X &= (I + A + A^2 + A^3 + \dots + A^n) \cdot \Delta Y \\ \Delta X &= I \cdot \Delta Y + A \cdot \Delta Y + A^2 \cdot \Delta Y + A^3 \cdot \Delta Y + \dots + A^n \cdot \Delta Y \end{aligned} \quad (9)$$

Equation (9) is equivalent to equation (6) and shows us that by increasing the final demand of a sector j , the economy will respond by increasing the output of that sector. The increase in the total production of this sector corresponds to the variation in demand from the other sectors plus the value of inputs needed to produce, which corresponds to the first two terms on the right side of the equation (9). However, all other sectors supplying inputs must increase production to meet the final demand. The value of this demand is represented by the term, and this will occur successively. $\Delta X A^2 \cdot \Delta Y$

Therefore, it can be concluded that the L matrix encompasses the direct and indirect impacts of the variation in the final demand of the economy as a whole (Guilhoto, 2011, Moore & Petersen, 1955).

THE MULTIREGIONAL INPUT-OUTPUT MODEL – MRIO

From now on, MRIO, the multi-regional IO model, seeks to expand the intersectoral relationships presented to include transactions between sectors from different regions (Yamano, 2017; Leontief & Strout, 1961), previously treated exogenously to the model. The MRIO model explains how the products and services of the industries of one region are absorbed by the industries or the final demand of other regions (Leontief; Strout, 1961).

One of the significant differentials of the MRIO is that flows of goods between the sectors of the different regions, previously treated as imports, are treated endogenously to the model, including sectoral exchanges at the regional level in the I-O representation (Miller & Blair, 2009).

The process consists of isolating two or more regions from the rest of the economic system, in order to analyze the relationship between them internally to the model (Leontief; Strout, 1961, Miller & Blair, 2009). The initial applications of the methodology were dedicated to studying regions at the subnational level (Isard & Kuenne, 1953, Miller, 1957, Hirsch, 1959), and later expanded to international application.

Miller and Blair (2009) point out that the replacement of the term "region" by "nation" is possible within the framework of the MRIO, as will be demonstrated in this section, and comes as a consequence of the growing economic interdependence of nations. Therefore, this section seeks to support the

⁹ Here, n refers to the number of "layers" of the multiplicative effects of final demand on the total output of the other sectors, and since the indirect effects occur in a chain and in an infinite way, it is possible to make n tend to infinity.

multiregional input-output analysis from an international perspective, which is the object of study of this work.

A systematic review of methodologies at the subnational level and how the transition to global models occurred is beyond the scope of this work and can be found in Miller and Blair (2009), Guilhoto (2011) and Isard and Langford (1971). The functioning of the model depends on the availability of data for inter- and intra-regional sectoral transactions (Miller and Blair, 2009).

Importantly, these models often contain thousands and even millions of observed data¹⁰, even in scaled-down and simplified models. According to Yamano (2017), there is no other way to macroeconomically analyze this amount¹¹ of data.

Figure 3 shows a simplified diagram of I-O relationships in an interregional system with two regions, L and M, where trade relations for intermediate inputs are known. The sectors of the L and M regions can be both suppliers and buyers of inputs, as well as there may be final demand from the L and M regions for the finished products of the sectors of both regions (Guilhoto, 2011). The total output of the sectors of the two regions is treated separately, and the flows from the other regions entering the system are considered as imports from the rest of the world.

	Setores - Região L	Setores - Região M	L	M	
Set. Reg. L	Insumos Intermediários LL	Insumos Intermediários LM	DF LL	DF LM	Prod. Total L
Set. Reg. M	Insumos Intermediários ML	Insumos Intermediários MM	DF ML	DF MM	Prod. Total M
	Imp. Resto Mundo (M)	Imp. Resto Mundo (M)	M	M	M
	Impostos Ind. Liq. (IIL)	Impostos Ind. Liq. (IIL)	IIL	IIL	IIL
	Valor Adicionado	Valor Adicionado			
	Prod. Total Região L	Prod. Total Região M			

Figure 3 Input-output relationships in an interregional system. Source: Guilhoto (2011)

¹⁰ As an example, according to Yamano (2017) a model with 20 industries, 50 countries and 5 components of final demand will contain 1,255,000 observed data.

¹¹ A major obstacle to input-output analysis is calculating the term $(I-A)^{-1}$, which, depending on the size of the matrix $(I-A)$, can demand a very high computational capacity. Miller and Blair (2009) mention that this was a challenge overcome as computing advanced. "In 1939, it took 56 hours to invert a matrix [input-output] of 42 sectors [on Harvard's Mark II computer]. In 1947, it took 48 hours to invert an input-output matrix of 38 sectors. However, in 1953, the same operation took only 45 minutes." (Morgenstern, 1954 *apud* Miller and Blair, 2009, p. 31, footnote)

The Z intersectoral exchange matrix is made up of 4 matrix components:

$$Z = \begin{bmatrix} Z^{LL} & Z^{LM} \\ Z^{ML} & Z^{MM} \end{bmatrix} \quad (10)$$

Where:

Z_{ij}^{LL} – flows between the supplying sector i of region L to the purchasing sector J of region L, i.e. the intermediate demand of the sectors of region L.

Z_{ij}^{LM} – flows between the supplying sector i of region L to the purchasing sector J of region M, i.e. the interregional intermediate demand from the sectors of region L to region M.

Z_{ij}^{MM} – flows between the supplying sector i of region M to the purchasing sector J of region L, i.e. the intermediate demand of the sectors of region M.

Z_{ij}^{ML} – flows between the supplying sector i of region M to the purchasing sector j of region L, i.e. the interregional intermediate demand from the sectors of region M to region L.

The basic identity of the input-output models, presented in equation (1), can be expanded by considering the additional exchanges of the model (Miller & Blair, 2009). Note that the part of the final demand for the products of sector i of region L used as *inputs* of the production processes of sectors j of region M is now explained in the second term, summation of equations (11) and (12):

$$x_i^L = z_{i1}^{LL} + z_{i2}^{LL} + \dots + z_{ij}^{LL} + z_{i1}^{LM} + z_{i2}^{LM} + \dots + z_{ij}^{LM} + y_i^{LL} + y_i^{LM}$$

Written in the generalized way, we have:

$$x_i^L = \sum_{j=1}^n z_{ij}^{LL} + \sum_{j=1}^n z_{ij}^{LM} + y_i \quad (11)$$

$$x_i^M = z_{i1}^{ML} + z_{i2}^{ML} + \dots + z_{ij}^{ML} + z_{i1}^{MM} + z_{i2}^{MM} + \dots + z_{ij}^{MM} + y_i^L + y_i^M$$

Or, more generally:

$$x_i^M = \sum_{j=1}^n z_{ij}^{ML} + \sum_{j=1}^n z_{ij}^{MM} + y_i \quad (12)$$

Where:

x_i^L – total production of sector i of region L

x_i^M – total production of sector i of region M

y_i^L – final demand from region L for the products of sector i

y_i^M – final demand from region M for sector i products

Analogous to equation (10), the total output will be:

$$X = \begin{bmatrix} X^L \\ X^M \end{bmatrix} \quad (13)$$

Where:

$$X^L = \begin{bmatrix} x_1^L \\ x_2^L \\ x_3^L \\ x_4^L \\ \vdots \\ x_i^L \end{bmatrix} \quad (14) \quad \text{and} \quad (15) \quad X^M = \begin{bmatrix} x_1^M \\ x_2^M \\ x_3^M \\ x_4^M \\ \vdots \\ x_i^M \end{bmatrix}$$

And the final demand vector will be:

$$Y = \begin{bmatrix} Y^L \\ Y^M \end{bmatrix} \quad (16)$$

Where:

$$Y^L = \begin{bmatrix} y_1^L \\ y_2^L \\ y_3^L \\ y_4^L \\ \vdots \\ y_i^L \end{bmatrix} \quad (17) \quad \text{and} \quad Y^M = \begin{bmatrix} y_1^M \\ y_2^M \\ y_3^M \\ y_4^M \\ \vdots \\ y_i^M \end{bmatrix} \quad (18)$$

Matrix A of technical production coefficients is defined, which will have four matrix components:

$$A = \begin{bmatrix} A^{LL} & A^{LM} \\ A^{ML} & A^{MM} \end{bmatrix} \quad (19)$$

Each sub-matrix of A will have its technical coefficient of production a_{ij} according to sectoral exchanges and total production of sectors in regions L and M:

$$A^{LL} \rightarrow a_{ij}^{LL} = \frac{z_{ij}^{LL}}{x_j^L} \quad (20)$$

$$A^{LM} \rightarrow a_{ij}^{LM} = \frac{z_{ij}^{LM}}{x_j^M} \quad (21)$$

$$A^{MM} \rightarrow a_{ij}^{MM} = \frac{z_{ij}^{MM}}{x_j^M} \quad (22)$$

$$A^{ML} \rightarrow a_{ij}^{ML} = \frac{z_{ij}^{ML}}{x_j^L} \quad (23)$$

Where:

A^{LL} – how much the sectors of region L demand from the sectors of region L to generate one dollar in their total production, that is, the matrix of technical coefficients of production of region L.

A^{LM} – how much the sectors of region L demand from the sectors of region M to generate one dollar in their total production, that is, the matrix of technical coefficients of production between region L and M.

A^{MM} – how much the sectors of region M demand from the sectors of region M to generate one dollar in their total production, that is, the matrix of technical coefficients of production of region M.

A^{ML} – how much the sectors of region M demand from the sectors of region L to generate one dollar in their total production, that is, the matrix of technical coefficients of production between region M and L.

By replacing the z_{ij} coefficients of (11) and (12) with the a_{ij} coefficients of equations (20) to (23), we get:

$$x_i^L = a_{i1}^{LL} \cdot x_1^L + a_{i2}^{LL} \cdot x_2^L + \dots + a_{ij}^{LL} \cdot x_j^L + a_{i1}^{LM} \cdot x_1^M + a_{i2}^{LM} \cdot x_2^M + \dots + a_{ij}^{LM} \cdot x_j^M + y_i^L \quad (24)$$

$$x_i^M = a_{i1}^{ML} \cdot x_1^L + a_{i2}^{ML} \cdot x_2^L + \dots + a_{ij}^{ML} \cdot x_j^L + a_{i1}^{MM} \cdot x_1^M + a_{i2}^{MM} \cdot x_2^M + \dots + a_{ij}^{LM} \cdot x_j^M + y_i^L \quad (25)$$

Solving in Y (24) and (25) and placing the components in matrix form, we get:

$$Y^L = (I - A^{LL}) \times X^L - A^{LM} \times X^M \quad (26)$$

$$Y^M = -A^{ML} \times X^L + (I - A^{MM}) \times X^M \quad (27)$$

Equation (28) is nothing more than the aggregate form of matrix equations (26) and (27). Note that, considering the definitions of equations (10), (13), and (16), equation (28) is structurally equal to equation (5), which is the basic input-output model.

$$\begin{bmatrix} Y^L \\ Y^M \end{bmatrix} = \left\{ \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} A^{LL} & A^{LM} \\ A^{ML} & A^{MM} \end{bmatrix} \right\} \begin{bmatrix} X^L \\ X^M \end{bmatrix} \quad (28)$$

Thus, the multi-regional input-output model consists of an extension of the basic input-output model to encompass multiple regions within the model. Although this section has represented a two-region model, it is possible to expand the model to n regions without changing its structure (Miller & Blair, 2009). International I-O databases, including the EE-MRIO, provide consistent data on sectoral relations between countries, address environmental concerns, and extend the MRIO to economic and environmental studies.

THE ENVIRONMENTALLY EXTENDED MULTIREGIONAL INPUT-OUTPUT MODEL – EE-MRIO

The EE-MRIO is an extension of MRIO that identifies how economic vectors of final demand drive environmental impacts in production processes. It helps in correctly identifying and allocating the

environmental impact to each sector in its buying and selling relationships, using CO₂ emissions as an example (Kitzes, 2013).

Let $E_{i,i}$ be the total emissions of a sector i over the course of any given year. Let also $x_{t,i}$ be the total output of sector i over the same year. In an I-O model with n sectors, the d -line vector of emissions intensity is defined as the ratio of total emissions to total output of sector i :

$$d(i) = \frac{E_i}{x_i} \quad (29)$$

By pre-multiplying the emission intensity vector d by equation (5), it is possible to calculate the emission vector F , which reports the total emissions of the economy caused by the final demand Y (Kitzes, 2013):

$$F = d \times (I - A)^{-1} \times Y \quad (30)$$

The intensity vector F shows how the total emissions from the production processes move through the sectors of the economy, allocating them in a consistent manner (Kitzes, 2013). The vector $d \times (I - A)^{-1}$ shows the total *upstream emissions* of each of the n sectors in the production of one dollar to meet the final demand.

METHODOLOGICAL APPROACH TO CIRCUMVENT THE DOUBLE-COUNTING PROBLEM IN MRIO

The complete analysis of the impacts of the life cycle of materials depends on the ability to distinguish between the sectors that supply the inputs necessary for the production process (upstream sectors), as well as the industry sectors and the final demand they use for the goods and services produced (downstream sectors) (Dente *et al.*, 2018). The analysis may suffer from the problem of double counting in intermediate goods and services, due to the principle of circularity of the economy.

In order to circumvent this problem, Dente *et al.* (2018) propose to highlight from within the input-output model materials to be analyzed, which are produced by sectors henceforth referred to as target sectors, or TS (*target sectors*), while materials that are not in focus are produced by those from Non-Target Sectors, or NTS (*non-target sectors*).

Figure 4 shows that the sectors supplying inputs to the target sectors are located in the upstream phase of the economy, while the sectors or final demand that consume the materials under analysis are located in the downstream phase¹².

¹² By definition, the downstream phase contains only the non-target sectors and the final demand, since the self-consumption of the target sectors is allocated in the upstream phase as supply to the production processes of the analyzed materials.

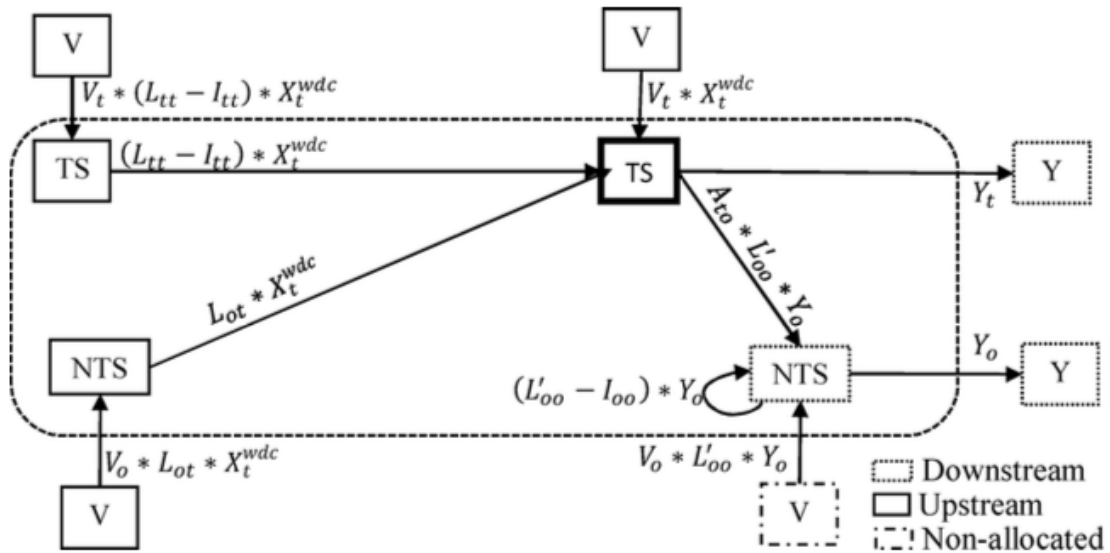


Figure 4 Schematic model of the upstream and downstream flows of the target sectors. Source: Tooth et. AL (2018)

The purpose of this classification is to identify a total production, without double counting (X_{twdc}), for which by applying the Leontief framework in the supply chains of the target sectors (L_{tt}) represents their total production (X_t), namely:

$$X_t = L_{tt} * X_t^{wdc} \tag{31}$$

The amount of output of the target sectors that occurs when directly applying the input-output structure to the model should be removed from the total output, and can be explained by:

$$X_t^{dc} = X_t - X_t^{wdc} \tag{32}$$

Substituting (31) for (32) gets:

$$X_t^{dc} = X_t - (L_{tt})^{-1} * X_t \rightarrow X_t^{dc} = (I_{tt} - L_{tt}^{-1}) * X_t \tag{33}$$

It is necessary to define what will be the inverse matrix of intersectoral exchanges between the target sectors (L_{tt}). The sectoral relations of the matrices of interdependence coefficients decompose into four submatrices¹³, and, consequently, the Leontief inverse has the same structure in four submatrices.

The choice of the model to differentiate the target sectors from the non-target sectors implies greater complexity when considering the impacts upstream of the value chain, as shown in Figure 5, which presents the impacts of final demand on the economy up to the 3rd level (Dente *et al*, 2018).

¹³ A tt, to, ot and oo. Explain what each one is

S.M.R. Dente et al.

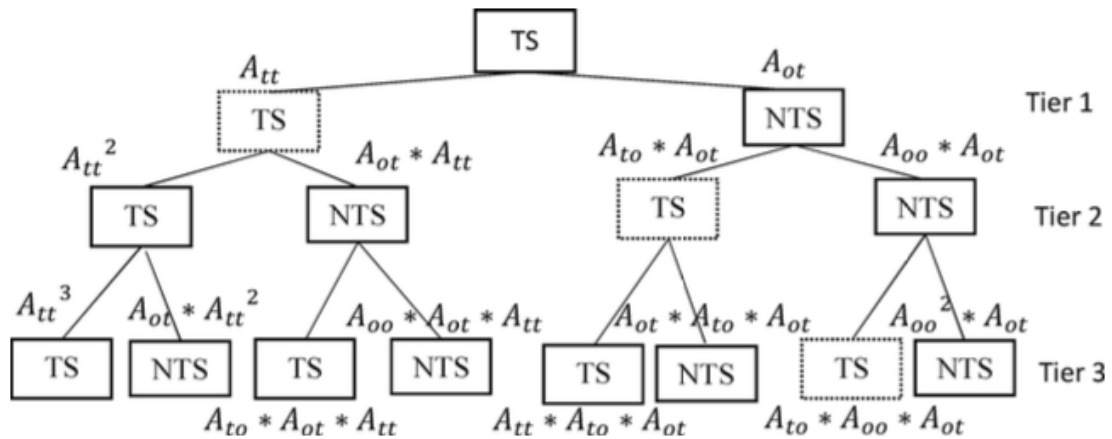


Figure 5 I-O model layers with the separation into target sectors (TS) and non-target sectors (NTS). Source: Dente *et al* (2018)

Contrary to Leontief's model, which makes a complete decomposition of supply chains, Dente *et al.* (2018) decompose the supply chain of the non-target sectors and keep the supply chain of the target sectors intact. Adding up the elements of the target sectors, represented by the dotted boxes in Figure 5, to obtain the total effect of the target sectors on the economy, we have:

$$A'_{tt} = A_{tt} + A_{to} * A_{ot} + A_{to} * A_{oo} * A_{ot} + A_{to} * A_{oo}^2 * A_{ot} + \dots \quad (34)$$

It is possible to highlight the terms Ato and Aot to obtain:

$$A'_{tt} = A_{tt} + A_{to} * (I_{oo} + A_{oo} + A_{oo}^2 + \dots) * A_{ot} \quad (35)$$

The infinite series in parentheses is nothing more than Leontief's inverse of the non-target sectors (L'oo). Therefore, we can rewrite equation (35) as follows:

$$A'_{tt} = A_{tt} + A_{to} * L'_{oo} * A_{ot} \quad (36)$$

The Leontief inverse of the target sectors (Ltt) will have a portion corresponding to the intersectoral exchanges of the target sectors (Att) and a portion corresponding to the decomposition of the value chain of the non-target sectors:

$$L_{tt} = (I_{tt} - A'_{tt})^{-1} \quad (37) \quad (38)$$

$$L_{tt} = (I_{tt} - (A_{tt} + A_{to} * L'_{oo} * A_{ot}))^{-1}$$

Inserting equation (38) into equation (33) yields:

$$X_t^{dc} = (A_{tt} + A_{to} * L'_{oo} * A_{ot}) * X_t \quad (39)$$

Once you have defined the double-counting value to be taken from the total output, you can use equation (32) to define the total output without double-counting. It is then possible to calculate the total production in the upstream and downstream phases of the value chain from the perspective of the materials produced by the target sectors:

$$X_o^{up} = L_{ot} * X_t^{wdc} \quad (40)$$

$$X_t^d = X_o - X_o^{up} \quad (41)$$

Through the partial decomposition of the value chain, therefore, it is possible to arrive at the submatrices that make up the Leontief inverse:

$$L_{tt} = I_{tt} + L_{tt} * (A_{tt} + A_{to} * L'_{oo} * A_{ot}) \quad (42)$$

$$(44) \quad (45)$$

$$L_{ot} = L'_{oo} * A_{ot} + L_{ot} * (A_{tt} + A_{to} * L'_{oo} * A_{ot})$$

$$L_{to} = L_{tt} * A_{to} * L'_{oo}$$

$$L_{oo} = L'_{oo} + L_{ot} * A_{to} * L'_{oo}$$

Rewriting equations (31) and (40) from the perspective of the final demand, and inserting equations (42) to (45), we have:

$$X_t = L_{tt} * Y_t + L_{to} * Y_o \quad (46)$$

$$(47)$$

$$X_t = L_{tt} * (Y_t + A_{to} * L'_{oo} * Y_o)$$

$$X_t^{wdc} = Y_t + A_{to} * L'_{oo} * Y_o$$

$$X_o = L_{ot} * Y_t + L'_{oo} * Y_o$$

$$(49) \quad (50)$$

$$(51)$$

$$X_o = L_{ot} * (Y_t + A_{to} * L'_{oo} * Y_o) + (L_{oo} - A_{to} * L'_{oo}) * Y_o$$

$$X_o = X_o^{up} + X_o^d$$

Equation (48) shows that the total output of the target sectors is the sum of the final demand and the share of the final demand of the target sectors needed to meet the final demand of the non-target sectors. Equation (50) shows that the total output of the non-target sectors is composed of two parts: one located upstream of the target sectors and the other located downstream of the target sectors. By dividing the supply chain into upstream and downstream of target sectors, the problem of circularity of the economy is circumvented by circumventing double counting.

The analysis of environmental impacts can be easily derived from the methodology of Dente *et al.*, (2018) by replacing the economic input-output structure with a structure of environmental stressors (S), through a vector of environmental impact of the activities of the target sectors (Dt) and the non-target sectors (Do):

$$S_t = D_t * L_{tt} + D_o * L_{ot} \quad (52)$$

$$S_o = D_t * L_{to} + D_o * L_{oo} \quad (53)$$

$$E_t^{up} = E_{tt}^{up} + E_{ot}^{up} = D_t * L_{tt} * X_t^{wdc} + D_o * L_{ot} * X_t^{wdc} \quad (54)$$

The total impacts in the downstream phase of the chain (E^d) – equation (55) – originate from the demand for primary inputs from the non-target sectors (E_{vd}) as well as from the inputs from the target sectors (E_{td}). Thus, an allocation of these total impacts (E^d) is necessary:

$$E^d = D_o * X_t^{wdc} \quad (55)$$

$$E^d = E_t^d + E_v^d \quad (56)$$

$$E_t^d = A_{to} * L'_{oo} * E^d \quad (57)$$

$$E_v^d = \text{diag}(V_o) * L'_{oo} * E^d \quad (58)$$

The research by Dente *et al.*, (2018, 2019) was applied to the Japanese economy and allowed to consistently analyze the most emitting sectors and correctly allocate the origin of these emissions upstream or downstream of the value chain. Section 3.2 will show how Cabernard *et al* (2019) deepened the method of Dente *et al.* (2018, 2019) to show the relationships without double-counting at the sectoral level.

DISCUSSION

The methodology of Dente *et al.*, (2018) is applicable for economic analyses at the national level. Cabernard *et al.*, (2019) delved into the methodology for analyzing the impact of embedded flows from *global supply chains*.

It is important to highlight that the methodology is applicable to any global input-output database (Cabernard *et al.*, 2019), and that the choice of this work to work with EXIOBASE was made due to its comprehensive sectoral resolution, while its regional resolution explicitly treats both Brazil and China as regions.

Figure 6 presents a conceptual scheme of the methodology, which consists of highlighting from the database the economic sectors within regions to be studied, similar to what was proposed by Dente *et al.* (2018), treating them as target sectors. The rest of the economy is treated as non-target sectors-regions, and their impacts will be allocated in the upstream and downstream phase of the value chain of the target sectors-regions, according to their role as supplier or demander of the goods and services in the supply chain of the target sectors-regions, respectively.

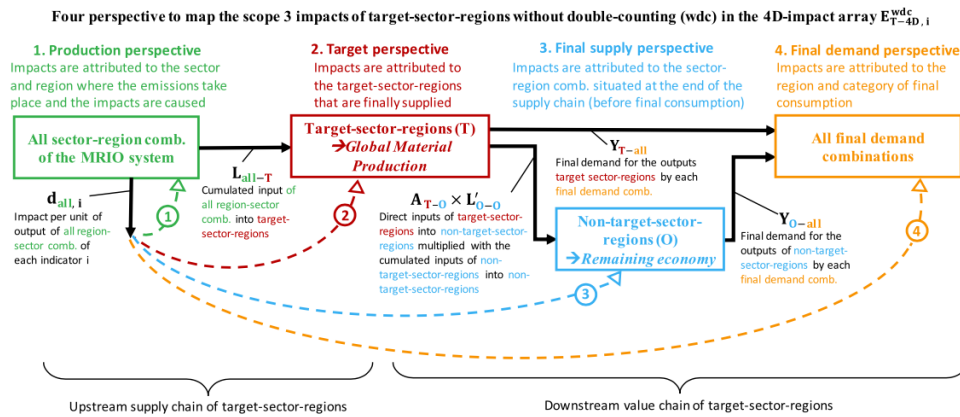


Fig. 1. Illustration of the methodology to assess the scope 3 impacts of target-sector-regions (here illustrated with the example of global material production) without double-counting. The four perspectives are connected in the four-dimensional (4D) impact array ($E_{T-4D,i}^{wdc}$), which allows to track the scope 3 impacts of target-sector-regions over four stages of the global value chain.

Figure 6 Structure of the allocation of environmental impacts according to the perspective of responsibility for the impact generated. Source: Cabenard *et al.* (2019).

The impacts are analyzed through the flows between 4 perspectives:

- (a) Producer's perspective: the direct impacts are attributed to the industrial sectors of the countries that carried out the production of the good or service.
- (b) Target perspective: Upstream impacts are attributed to the target sectors-regions that demand the goods and services, as well as the impacts of the production processes of the target sectors-regions.
- (c) Final supplier perspective: Impacts are allocated to the sectors at the end of the supply chain, i.e. the sectors that produce the final products.
- (d) Final demand outlook: allocates the impacts on the different categories of final demand that consumed the goods and services of the target sectors-regions.

The methodology of Cabenard *et al.* (2019) makes it possible to measure not only each perspective in the value chain, but also how the different perspectives talk to each other, that is, how the different agents transfer the responsibility for their environmental impacts to each other through flows embedded in the goods and services traded.

Let A be the matrix of coefficients of interdependence of the globalized input-output model. Let also v_t be a line vector representing the indices of the target sectors-regions, and v_o a line vector representing the indices of the non-target sectors-regions within the matrix A . You can define the A_{t_o} and A_{o_t} matrix as sub-matrices of the matrix A through the indices v_t and v_o :

$$A_{t_o} = A(v_t, v_o) \tag{59}$$

$$A_{o_t} = A(v_o, v_t) \tag{60}$$

The total output of the model can be calculated from equation (61). The top bar notation means the sum by rows of the different components of the final demand so that the vector becomes a column vector.

The final demand of all regions for the products of the target and non-target sectors-regions are defined in a similar manner, according to equations (62) and (63), respectively. The colon is used to identify the selection of all final demand categories from all regions. Similarly, X_t , L_{all-t} are calculated according to equations (64) and (65):

$$X = \overline{L} \times \overline{Y} \quad (61)$$

$$Y_{t-all} = Y(v_t, :) \quad (62)$$

$$Y_{o-all} = Y(v_o, :) \quad (63)$$

$$x^t = x(v_t) \quad (64)$$

$$L_{all-t} = L(:, v_t) \quad (65)$$

Let d_{all-i} be the vector-line containing the coefficients of the impact i for each unit of total output, for all sectors and regions of the MRIO model:

$$d_{all-i} = \frac{F(i,:)}{X} \quad (66)$$

Equations (67) to (70) represent the impacts of the value chain across the different perspectives, as shown in Figure 32. The term "diag" symbolizes the diagonalized vector, while the slash above it symbolizes a line-by-line sum of the elements.

All impacts of the non-target sectors-regions supplying inputs of the target sectors-regions are allocated upstream of the supply chain, channeled through the target sectors-regions, and finally will be allocated to the non-target sectors-regions that receive the intermediate or final outputs (Cabernard *et al.*, 2019)

$$E_{produtor-alvo} = \frac{diag(d_{all,i}) \times L_{all-t} \times}{diag(Y_{t-all} + A_{t-o} \times L'_{oo} \times Y_{o-all})} \quad (67)$$

$$E_{alvo-fornecedor\ final} = \frac{diag(d_{all,i} \times L_{all-t}) \times A_{t-o} \times L'_{oo} \times}{diag(Y_{o-all})} \quad (68)$$

$$E_{alvo-demanda\ final} = \frac{diag(d_{all,i} \times L_{all-t}) \times}{diag(Y_{t-all} + A_{t-o} \times L'_{oo} \times Y_{o-all})} \quad (69)$$

$$E_{produtor-demanda\ final} = \frac{diag(d_{all,i}) \times L_{all-t} \times}{diag(Y_{t-all} + A_{t-o} \times L'_{oo} \times Y_{o-all})} \quad (70)$$

Equations (67-70) circumvent the double-counting problem by allocating upstream of the chain the share of the total output of the target sectors-regions used as inputs for other target sectors-regions (Cabernard *et al.*, 2019), which was previously counted both downstream and upstream.

IMPLICATIONS

There are more than 90 methanol production plants in the world. As far as distribution is concerned, methanol can already be found in 100 ports. Data indicates that in 2020 Asia Yadav *et al.* (2020) (Irena, 2021)¹⁴ was the largest producer of methanol (40%), and the smallest, Europe (2%). According to, all the methanol used in Brazil is still imported, with 32% of the total directed to the production of biodiesel. (MMSA, 2021)

Although methanol is mostly produced from fossil sources, the components necessary for its production can be obtained from various types of raw materials. Methanol is generated through syngas, which consists of a mixture of hydrogen, carbon dioxide, and carbon monoxide. When produced from biomass, in a sustainable way, it can considerably reduce the environmental impacts generated by fossil fuels. (Blumberg *et al.*, 2019; Poudyal, *et al.*, 2016; Yadav, A., *et al.*, 2020)

Bio methanol can be produced from the gasification of lignocellulosic biomass, such as forest residues, agricultural residues, among other sources of organic matter, as well as through biogas, which can be obtained from the bio digestion of animal waste, agro-industrial waste, sanitary sewage or landfills. (Müller-Casseres, *et al.*, 2020; Oliveira *et al.*, 2020)

Figure 7 shows that it is possible to notice that, while in the first route the syngas is obtained through the reforming of biomethane, in the second it occurs via biomass gasification.

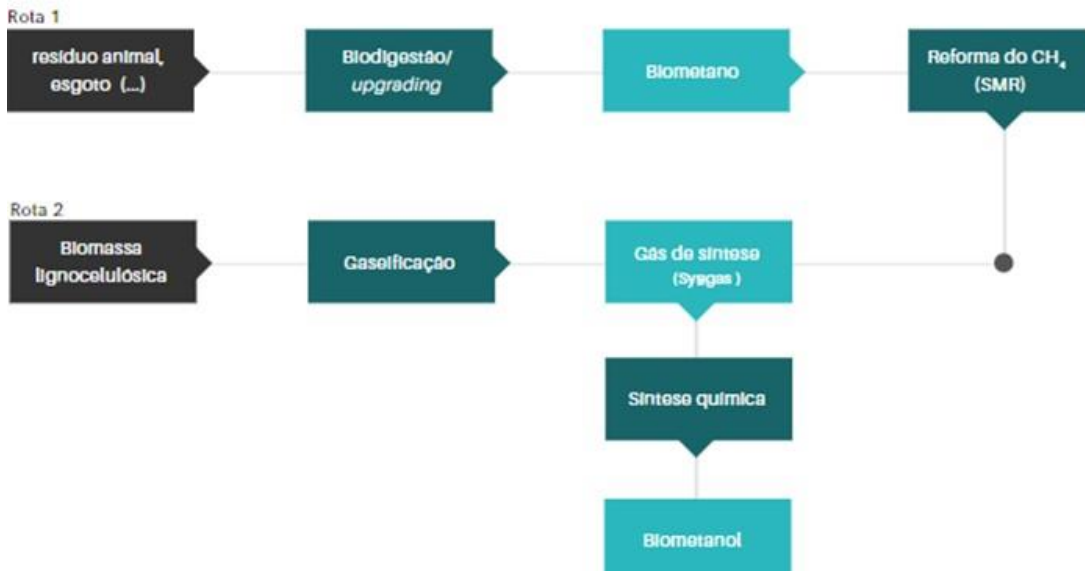


Figure 7 Biomethanol production routes, adapted from Carvalho *et al.*, 2021)

It is also possible to produce methanol from obtaining hydrogen via the electrolysis process with the use of renewable energy, along with obtaining CO₂ via air capture or exhaust from biomass combustion,

¹⁴ Not including China

generating renewable methanol. This makes it possible to produce syngas, and subsequently CH₃OH. (Hobson & Marquez, 2018; Irena, 2021; Oliveira *et al.*, 2020)

Methanol, a global commodity, is chemically identical to fossil fuels and is produced by biomass. Renewable methanol production is less than 0.2 Mt annually, with most coming from biomass. To meet IMO targets, carbon-neutral fuels need to contribute 40% of the shipping sector's fuel mix by 2050. The future marine fuel market will have different energy sources and regional integration. BioLNG, bioMGO, and biomethanol are strong candidates in areas with sufficient biomass availability, while e-fuels and fossil fuels become more competitive. (Irena, 2021; Markopoulos *et al.*, 2022; DNV, 2022). In sum, this paper reviewed the development of environmentally extended multi-regional input-output matrices (EE-MRIO) through a literature review. It provides insights for scholars, decision-makers, and practitioners on the evolution of these models. This work has also implications in other fields of research, such as: (a) oil & gas companies (Aylmer *et al.*, 2024a; 2024b); (b) project management (Dias *et al.*, 2022; Soares *et al.*, 2020); (c) civil work projects (Dias, 2016), among others.

FUTURE STUDIES

Future researchers are encouraged to pursue the EE-MRIO, developed by Wassily Leontief. This model reveals the interdependence of industries in an economy. The I-O model, also known as "general-equilibrium models," shows the relationships between sectors that produce outputs and consume inputs in their production process. The relationships are expressed through two-dimensional intersectoral transaction tables.

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