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# Effect of Markov Chain and System of Linear Equations on the Analysis of Nigerian Current Account Net

# <sup>1</sup>Azor, P. A., <sup>2</sup>Gilobeni, E.P. and <sup>3</sup>Amadi, I.U.

<sup>1</sup> and 2Department of Mathematics & Statistics, Federal University, Otuoke, Nigeria, <sup>2</sup>Department of Mathematics & Statistics, Captain Elechi Amadi Polytechnics, Port Harcourt, Nigeria

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**ABSTRACT:** The idea behind Markov chain model and system of equations cannot be emphasized in terms of modeling physical systems like the Nigerian Current Account formation; since each finite state communicates for suitable decisions making. So this paper studied stochastic analysis of Markov chain on NCA data (2004-2022). The NCA data were transformed into 3-steps transition probability matrix solution to cover the number of independent years. From the transition matrix of stochastic analysis showed that in 2004-2012 has the highest probability of reducing by 66% and finally in 2014-2022 has the highest probability of no-change in payments of goods and services by 3.3%. The future NCA data changes were known by introducing the concepts of percentages in each interval of years as column vectors where system of linear equations were developed and solutions were obtained to guide Nigerian economy on various levels of imports and exports of goods and services. Lastly, other statistical disparities were also considered and well discussed accordingly. These results are informative to Nigerian economy to effectively take vital decisions in their investment plans.

KEYWORDS: NCA, stochastic analysis, Markov chain, system of equations, transition matrix

# **INTRODUCTION**

Current Account is an economic concept which measures a country's transactions with the rest of the globe, characteristically, its net trade in goods and services, its net revenue or returns or transnational investments and its net transfer payments, over a finite interval, such as a quarter or a year. It is the nation's trade balance or the balance of imports and exports of goods and services, plus returns or foreign investments minus payments to foreign investors or nations. The current account is one of the three components of a nation's balance of payment system which serves as

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standard or measure for all internationally transferred Capital. The other two components of a nation's balance of payment system are the capital account and the financial accounts.

All nations have the target of building a trade surplus, where their goods and services are expected in order to increase their revenue base. The current account is considered to be balanced, when a nation can produce for her needs. However, it is unbalanced, when more goods and services are imported than exported, resulting to a deficit, since more revenue is paid to foreign investors or nations. The current account is pivotal in the long-term economic growth, particularly in a developing nation like Nigeria, especially, considering the significance of trade as a major source of foreign exchange returns. Some factors such as, advancement in information and Communication technology, global paradigm shift to trade, boom in commodity price, increase in trade openness and the increasing role of developing nations in the global economy [1], have brought about a drastic increase of Nigeria dependence on international trade.

Nigeria current account predicament is source of worry to policy makers. Because the Nigerian economy is driven by external sector, which depicts the dependence of the economy on external sector to generate foreign exchange to import capital goods for increased economic activities in the real sector.

Moreover, money from the export of crude in particular, makes up over 60% of government's revenue, in recent times. Hence, the capacity of government to provide basic infrastructures like electricity, road, water, education, etc, is connected directly to the performance of the external sector of the economy.

Markov Chain or Markov Process is a sequence of experiments consisting of a finite number of states having some known probabilities,  $P_{ij}$ , where,  $P_{ij}$  is the probability of moving from State j or simply put is Stochastic Process which is dependent on immediate outcome and not on its history.

A Markov Chain may be considered as a series of transitions between different States, such that the probabilities associated with each transition is dependent only on the immediate proceeding state and not on how the process arrived at that state and the probabilities associated with the transitions between the states are constant with time.

Any variable whose value changes over a period of time in an uncertain way is considered to follow a Stochastic Process. A Markov Chain or a Markov Process is a particular type of Stochastic Process where only the present value of a variable is relevant for the prediction of the future. The past values of the variable and the way the present values of the variable have emerged from the past are irrelevant. Stock prices are usually assumed to follow Markov Process (Hull, 2018). A Markov Chain is relevant to the analysis of Stock prices in two ways:

(i) As a useful tool for making probabilistic statements about future stock price levels; and

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(ii) As an extension of the random walk hypothesis. Which constitutes an alternative to the more traditional regression forecasting methods to which it is in some unique way superior in the analysis of stock price behaviour.

Markov Chain model has been extensively applied in the prediction of stock market trends, which is very significant for investment decision. [2] applied Markov Chain model to historical share price data of a banking firm, HBL, and the result revealed that the data set exhibited the Markovian property and periodicity validated by convergence of transition probability matrix to a steady state distribution, which showed that Markov process model can be used for shares traded on Pakistan Stock Exchange. Similarly, [3] used a Markov Chain model in analyzing the movement of stock market and also forecast its share prices. The result obtained revealed that the Markov Chain model is an statistical technique of prediction which analyzes and predicts the future behaviour of stock market through initial initial state probability vector. In another study on the application of Markov Chain, [4] applied Markov Chain to model and forecast trends of Dangote Cement shares trading in the Nigerian stock Exchange over a period covering 1<sup>st</sup> January,2018 – 31<sup>st</sup> December, 2019, which formed 464 days trading data panel. The study resulted to the determination of a Markov Chain model based on probability transition matrix and initial state vector. And in the long run, irrespective of the current state of share price, the model predicted that the Dangote Cement would depreciate, maintain and appreciate respectively. [6] used the Markov Chain model to analyze and to make predictions on the three states that exist in stock price change, which are share price, decreased or remain unchanged. The Guaranty Trust Bank and the First Bank, all in Nigeria, were used as the two top banks for illustration. The transition matrix was derived using the MS Excel. It was observed that regardless of a bank current share price, in the long run, it could be predicted that its share price will depreciate with a probability of 0.4229, remain unchanged with a probability of 0.2072 and appreciate with a probability of 0.3699. [7] examined stock price formation in finite states, through the stochastic analysis of Markov chain model, and the data were subjected to 5-step transition matrix for independent stocks, where transition matrix replicated the use of 3-states transition probability matrix. Out of the four stocks studied, i.e. stock(1), stock(2), stock(3) and stock(4), it was observed that stock(1) had the highest mean and rate of return: 4.0548 and stock(4) had the best probability of price increasing in the near future: 21%. The stochastic analysis showed that stock price changes are memory-less, which satisfied the properties of Markov Chain, that is, converging at a point or becoming stationary at; n=5, i.e. *S*1:0.1967-0.2354, *S*2:0.2053-0.1913, *S*3:0.1972-0.2051 and *S*4:0.2023-0.1835. [8] compared the performance of five popular stocks using Markov modelling. Result of the analysis based on three year monthly closing price, gave an insight into the future possibilities of these five stocks. Reliance was observed to have the highest futuristic probability and it was followed by BPCL. While Oil India and IOC had higher probability to remain stable without much fluctuation. HP showed the highest probability of a fall from the existing state. [9] examined the stochastic analysis of stock market prices of Dangote Cement and Bua Cement PLCS respectively, using Markov Chain formation of finite states. The stock price data of the two companies, were subjected to a 3-step transition probability matrix, while the main data of October –December of each year

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(i.e. 2017-2021), were computed and used as column vectors. It was observed from the stochastic analysis that Dango Cement PLC, had the highest rate of return: 137.4371 and with the best probability of price reduction in the near future: 34% and a 32% probability of no change in price. A lot of scholars have extensively written on Markov chains namely:[10-15] etc.

Finally, due to the instability on the value of goods and services on Nigerians earning and spending; this can be linked as stochastic formation. For that reason, the method of Markov chain was used to study the Nigerian Current Account net for the period of 2004-2022. The NCA data replicated 3-states transition probability matrix solution for each independent year and percentage changes was introduced and used as row vector to determine the effect of NCA movements. The future NCA data changes were known by introducing the concepts of percentages in each interval of years as column vectors where system of linear equations were developed and solutions were obtained to guide Nigerian economy on various levels of imports and exports of goods and services. From the stochastic analysis the means, standard deviations and other variations were obtained. Finally This paper extends the work of [10] by assessing the movements of NCA , stating their future stock price changes and incorporating system of equations on its analysis. This novel contribution is unique and effective in handling NCA net movements

This paper is arranged as follows: Section 2.1 presents mathematical Framework, Data Analysis, Results and Discussion are presented in Section 3.1 and concluded in Section 4.1.

# **2.1. Mathematical Framework**

Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

# **Definition 1: Markov Chain**

Let  $\{X_n, n = 0, 1, 2, ...\}$  be a stochastic process that takes on a finite number of possible values. If  $X_{n=i}$ , then the process is said to be in state i at time n. Given that P  $\{x_{n+1} = j/x_n = i, x_{n-1}, ..., x_0 = i_0\} = P_{ij}$ .

Anytime the process in state i, there is a fixed probability  $P_{ij}$  that the next state will be state j.

The equation above means that the conditional distribution of any future state  $X_{n+1}$  with past states  $X_0, X_1, \ldots, X_{n-1}$  and the present state  $X_n$  does not fully depend on the historical sequence of past states but depends strictly on the immediate preceding state.[10].

# **Definition 2:** A stochastic Process

 $X = \{X_n; n \ge 0\}$  on a countable set S is a Markov Chain if, for any I,  $j \in S$  and  $n \ge 0$ ,

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$$P\{X_{n+1} = j | X_0, ..., X_n\} = P\{X_{n+1} = j | X_n\}$$
(1)  

$$P\{X_{n+1} = j | X_n = i\} = Pij$$
(2)

$$P\{X_{n+1} = J | X_n = I\} = PIJ$$
(2)

The Pij is the probability that the Markov Chain jumps from state i to state j.

These transition probabilities satisfying  $\sum_{i \in S} P_{ii}$  is the transition matrix of the chain.

Since the state space is countable, we can lab. the states as integers, such as  $S = \{0, 1, 2, ...\}$ 

#### 2.1.2 Probabilities of Sample Paths

When analyzing the structure of a stochastic process, it is important to describe its finitedimensional distributions. We will show that a Markov Chain distribution  $X_n$  are products of its transition probabilities and the probability distribution of the initial state  $X_0$ .

**Proposition 1:** Given that  $X_n$  is a Markov Chain on S with transition probabilities  $P_{ij}$  and initial distribution  $a_i = P\{X_0 = i\}$ . Then, for any  $i_0, ..., in \in S$  and  $n \ge 0$ .  $P \{X_0 = i_0, ..., X_n = i_n\} = \propto_{i_0} P_{i_0}, i_1, ..., P_{i_{n-1}, i_n} P_{i_n}, i_{n+1}\}$ where P = { $X_{n+1} = x_{n+1}/A_n$ } = P<sub>in</sub>,  $i_{n+1}$  by the Markov Property.

This implies that the probability of the Markov Chain to traverse a path  $i_0, i_1, \ldots, i_n$  is the multiplication  $P_{to,i1, \dots}$   $P_{in-1,in}$  of the probabilities of these transitions.

Hence, the probability that the Markov Chain up to time n has a sample path in a subset P of  $S^{n+1}$ is

$$p\{(X_{0}, ..., X_{n}) \in P \\ = \Sigma P\{X_{0} = i_{0}\} P_{i_{0}, i_{1}, ..., n} P_{i_{n-1}, i_{n}}$$
(3)  
$$(i_{0}, ..., i_{n} \in P)$$

In this study, Xn represents the daily profits of one of the selected companies, then the probability there will be an increase that in profits in .... number of days is  $\mathsf{P}\left\{X_0 \le X_1 \le \dots \le X_n / X_0 = i_0\right\}$  $=\sum_{i=1}^{n} P_{io,i_1\dots}\sum_{i=1}^{n} P_{i_{n-1,i_n}}$ 

These we will do for all the selected companies, as most probabilities like these for the Markov Chain can be expressed in terms of the transition matrix  $P = (P_{ij})$  and its  $n^{\text{th}}$  product  $P^n$ ,  $n \ge 0$ ; By definition,  $P^0 = 1$  (identity matrix), and  $P^n = P^{n-1}$ , for  $n \ge 1$ .

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Given that  $P_{ij}^n$  is the (i, j) the entry of  $P^n$ , then by matrix multiplication,

$$P_{ij}^{n} = \sum_{i_{1},...,i_{n-1} \in s^{n-1}} P_{i,}i_{1}P_{i},i_{2} \dots P_{i_{n-1},j}$$
(4)

2.1.3: n-step probabilities

The probability  $P{X_n = j/X_0 = i}$  is the sum of the probabilities of all paths of the form  $i, i_1, ..., i_{n-1}, j$ .

Consequently,  

$$P{X_n = j/X_0 = i} = P_{ij}^n$$

This can be obtained by computing  $\mathbb{P}^n$ . We donate the initial distribution  $\propto i = \mathbb{P}\{X_0 = i\}$  as a row vector  $\propto = (a_i)$ , then we have  $\mathbb{P}\{X_n = j\} = (\propto \mathbb{P}^n)j$ , which is the j<sup>th</sup> value of the row vector  $\propto \mathbb{P}^n$ .

Then multiplication proparty of matrices  $P^{m+n} = P^m P$  for m, n = 1 yields the Chapman-Kolmogorov equations.

$$P_{ij}^{m+n} = \sum_{K \in S} P_{ik}^m P_{kj}^n, i, j \in S.$$

Thus, the probability of the chain moving from state i to j in m+n steps is equal to the probability that it moves from i to any  $K \in S$  in m steps, and then from K to j in n more steps[10].

# 2.1.4 Construction of Markov Chains

Proposition II: Given  $\{X_n : n \ge 0\}$  is a stochastic process on S of the form  $X_n = f(X_{n-1}, Y_n), n \ge 1$ (6)

Where f: Sx  $S^1 \rightarrow$  and  $Y_1, Y_2$ .. are independent and identically distributed random variables with values in a general space  $S^1$  that are independent of  $X_0$ . Then  $X_n$  is a Markov Chain with transition probabilities Pij = P{f(i, y, ) = j}.

**Theorem** 1. (Construction of Markov Chains) let Pij be Markovian transition probabilities, and let  $\propto$  be a probability measure on S. (S = {0, 1, ...}. Suppose U<sub>0</sub>, U<sub>1</sub>, ... are i.i.d with a uniform distribution on [0, 1].

Assume 
$$X_0 = h(U_0)$$
, where  $h(u) = j$ , if  $U \in Ij$ , for some  $j \in S$ . Define  $X_n = f(X_{n-1}, U_n), \ge 1$ ,  
(7)

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where for each i, f(i,u) = j, if  $U \in I_{ij}$ , for some  $J \in S$  and  $I_{ij} = \left(\sum_{k=0}^{j-1} P_{ik}\right)$ 

Then  $\{X_n : n \ge 0\}$  is a Markov chain with initial distribution  $\propto$  and transition probability Pij.

#### 2.1.5 State Probability Distributions

The average transition process of Markov Chain is based upon the initial state of the system and the transition probability matrix to understand the chain completely. Hence,  $\pi^{(1)} = \pi^{(0)} A$ 

Hence,  $\pi^{(n)} = \pi^{(n)} A^n$   $\pi^{(2)} = \pi^{(0)} A^2$  $\pi^{(n+1)} = \pi^{(n)} A^n = \pi^{(0)} A^{n+1}$ , for  $n \ge 1$ 

This shows that the state probability vector at (n+1) is the product of the initial probability vector and  $n + 1^{th}$  power of the one-step transition probability matrix.

Given  $\pi$  as the stable probability distribution, Then  $\pi = \pi A$  input and output steady vectors will be the same at the time of steady state probabilities or stationary probability distributions.

**Theorem 2:** Suppose P is a stochastic matrix which implies the following:

i) P has non-negative entries or 
$$P_{ij} \ge 0$$
 (ii)  

$$\sum_{j} P_{ij} = \sum_{j} P(X_{n+1} = j / X_n = i) = \sum_{j} P(X_1 = j / X_0 = i)$$

which is stationarity or point of convergence.

Proof: (i) each associated entry in P is a transition probability  $P_{ii}$  and being probability  $P_{ii} \ge 0$ .

(ii) 
$$\sum_{j} P_{ij} = \sum_{j} P(X_{n+1} = j / X_n = i) = \sum_{j} P(X_1 = j / X_0 = i)$$

Which is stationarity.

$$P(X_i \in S / X_0 = i) = 1.$$

# Theorem 3 :( Chapman-Kolmogorov Equations).

$$P_{ij(m+n)} = \sum_{r=i}^{n} P_{ir(m)} P_{rj(n)} \text{ Since } P_{m+n} = P_m P_n \text{ and so on } P_n = P^n \text{ the } nth \text{ power of } P.$$

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Proof:

$$\sum_{r} P(X_{m+n} = j, X_m = r / X_0 = i)$$
$$\sum_{r} P(X_{m+n} = j / X_m = i / X_0 = i) P(X_m = r / X_0 = i)$$

Using the following in probability rule:

 $P_{ij(m+n)} = P(X_{m+n} = j / X_0 = i)$ 

 $P(A \cap B / C) = P(A / B \cap C)P(B / C) \text{ and setting}$  $A = \{X_{m+n} = j\}, B = \{X_m = r\}, \text{and } C = \{X_0 = i\}$ 

Using Markov property yields

$$P_{ij(m+n)} = \sum_{r} P(X_{m+n} = j / X_m = r) P(X_m = r / X_0 = i)$$

$$\sum_{r} P_{rj(n)} P_{ir(m)}$$

$$\sum_{r} P_{1r(m)} P_{r1(n)}$$

Hence  $P_{m+n} = P_m P_n$  and so  $P_n = P^n$ , the power of P.

To obtain an estimates of the transition probability as follows

$$P_{ij} = P(X_{t} = j / X_{t-1} = i), \text{ for } j = 0, 1, 2, 3, \dots, N$$

$$P_{ij} = \begin{cases} P \text{ if } j = 1 + j \\ q = 1 - P \text{ if } j = i - j \\ 0 \text{ otherwise} \end{cases}$$

where k + 1 is the number of states.

$$n_{ij} = \sum_{i=1}^{n} P_{ij} \text{ for } i j=0,2,3 \\ \frac{n_{ij}}{n_i} \text{ for } i j=0,1,\cdots k$$
(8)

However, for k = 3 is an estimate of the transition matrix.

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However, for k = 3 is an estimate of the transition matrix.

$$\hat{P}ij(NCA)_{2004-2012} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix}$$
(9)

$$\hat{P}ij(\text{NCA})_{2005-2013} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix}$$
(10)

$$\hat{P}ij(\text{NCA})_{2014-2022} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix}$$
(11)

Setting i j = 0, 1, 2 for k = 3

Introducing time to see the effect of percentage changes on (9-11) gives the following:

$$(\text{NCA})_{2004-2012} \left( P_{ij} \right) = \left( t_1, t_2, t_3 \right) \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix} = \left( \beta_1, \beta_2, \beta_3 \right)$$
(12)

$$(NCA) - NET_{2004-2012} \left( P_{ij} \right)' = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$
(13)

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$$(\text{NCA})_{2005-2013} \left( P_{ij} \right) = \left( t_1, t_2, t_3 \right) \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix} = \left( \beta_1, \beta_2, \beta_3 \right)$$
(14)

$$(\text{NCA})_{2005-2013} \left(P\right)' = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$
(15)

$$(\text{NCA})_{2014-2022} \left( P_{ij} \right) = \left( t_1, t_2, t_3 \right) \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix} = \left( \beta_1, \beta_2, \beta_3 \right)$$
(16)

$$(\text{NCA})_{2014-2022} \left( P \right)' = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$
(17)

#### 2.1.6 Stationary Probability Distribution of Markov Chain

The stationary property of Markov Chain states that irrespective of its initial state, how the stochastic process evolves, if transition steps increase, then the transition probability of reaching to state j from state i will converge to some constant value. (Dar et al, 2022).

$$\frac{\lim_{n \to \infty}}{\lim_{n \to \infty}} \operatorname{Aij}(n) = \frac{\lim_{n \to \infty}}{\lim_{n \to \infty}} A^n = \pi_j$$

This is known as the steady state probabilities. The n-step transition probability matrix represents the behaviour of the chain after n steps.

#### 2.1.7 Developing Markov Chain Model for Stochastic Analysis of NCA Movement

For proper accuracy of Markov chain model for future events; it needs to be developed for prediction of NCA payment movement. The initial payment needs to be in three finite states as follows:

(18)

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**R:** denotes the probability of NCA payment reducing in near future, **I** : denotes the probability of NCA payment increasing in near future, **NO-change** : denotes the probability of NCA payment not changing in near future However, probability of transition matrix shows the proper explanation of Markov chain. Every element in the matrix communicates. In order to form three states of Markov process we need to have the following table below:

# **Table 1: Transition Probability Matrix**

State	1	2	3	Total of Row
1	<i>P</i> <sub>11</sub>	<i>P</i> <sub>12</sub>	<i>P</i> <sub>13</sub>	
2	<i>P</i> <sub>21</sub>	<i>P</i> <sub>22</sub>	<i>P</i> <sub>23</sub>	<i>T</i> <sub>2</sub>
3	<i>P</i> <sub>31</sub>	<i>P</i> <sub>32</sub>	<i>P</i> <sub>33</sub>	$T_3$

In each entry  $P_{ij}$  indicates the number of times a transition is made from one state *i* to state *j*. the transition matrix is computed by simply dividing every element in each row through the total of each row. Nevertheless, this dissertation studies NCA payment data collected from statistical bulletin.

**2.1.8** System of Linear Equations on the Analysis of Nigerian Current Account Net Combining (12-17) gives the following system of equations which will be used for the analysis of NCA independently, hence we have:

# NCA-NET:2004-2012

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$$a_{11}P_{1} + a_{12}P_{2} + a_{13}P_{3} + \dots + a_{1n}P_{n} = b_{1}$$

$$a_{21}P_{1} + a_{22}P_{2} + a_{23}P_{3} + \dots + a_{1n}P_{n} = b_{1}$$

$$\vdots$$

$$a_{m1}P_{1} + a_{m2}P_{2} + a_{m3}P_{3} + \dots + a_{1n}P_{n} = b_{n}$$
(19)

NCA-NET:2005-2013

$$a_{11}P_{1} + a_{12}P_{2} + a_{13}P_{3} + \dots + a_{1n}P_{n} = b_{1}$$

$$a_{21}P_{1} + a_{22}P_{2} + a_{23}P_{3} + \dots + a_{1n}P_{n} = b_{1}$$

$$\vdots$$

$$a_{m1}P_{1} + a_{m2}P_{2} + a_{m3}P_{3} + \dots + a_{1n}P_{n} = b_{n}$$
(20)

NCA-NET:2014-2022

$$a_{11}P_{1} + a_{12}P_{2} + a_{13}P_{3} + \dots + a_{1n}P_{n} = b_{1}$$

$$a_{21}P_{1} + a_{22}P_{2} + a_{23}P_{3} + \dots + a_{1n}P_{n} = b_{1}$$

$$\vdots$$

$$a_{m1}P_{1} + a_{m2}P_{2} + a_{m3}P_{3} + \dots + a_{1n}P_{n} = b_{n}$$
(21)

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Where  $a_{ij}$ , i = 1, 2, 3...m; j = 1, 2, 3...n are well known coefficient of the system and  $b_j$ , i = 1, 2, 3...m are known scalars,  $P_1, P_2, P_3...P_n$  are the unknown (variables).

#### 3.1 Analysis, Results and Discussion

The data for this paper is gotten from statistical bulletin .To demonstrate the Nigerian Current Account performances in finite states. The secondary data covers from 2004-2022 every independent year was used to form transition probability matrix.

NCA movements	Reducing(R) its imports and exports payments	Increasing(I) ) its imports and exports payments	No(N) change in its imports and exports payments	Row totals of NCA
R	331429.7	263295.7	376024	970749.4
Ι	186084.8	52304.3	19488.7	257877.8
Ν	39422.8	12655.4	44731.2	96809.4

 Table 1: Nigerian Current Account (NCA) (2004-2012)

#### Table 2: Nigerian Current Account (NCA) from 2005-2013

NCA movements	Reducing(R) ) its imports and exports payments	Increasing(I) ) its imports and exports payments	No(N) change )in its imports and exports payments	Row totals of NCA
R	3455650.31	3478374.82	4698047.08	11632072.21
Ι	4891744.45	2056326.3	704560	7652630.75
Ν	117037.3	242901.3	713023.9	1072962.5

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 Table 3: Nigerian Current Account (NCA) from from 2014-2022

NCA movements	Reducing(R) ) its imports and exports payments	Increasing(I) ) its imports and exports payments	No(N) change in ) its imports and exports payments	Row totals of NCA
R	2064890.16	1970592.13	1641463.22	5676945.51
Ι	2736448.26	2996626.99	687906.39	6420981.64
Ν	-3033484.84	3174745.44	1630072.89	1771333.49

# Transition probability matrix for Nigerian Current Account (NCA) from 2004-2012

	0.3414	0.2712	0.3874
$NCA - NET_{2004-2012}(P) =$	0.7216	0.2028	0.07557
	0.4072	0.1307	0.4621 )

# Transition probability matrix for Nigerian Current Account (NCA) from 2005-2013

$$NCA - NET_{2005-2013}(P) = \begin{pmatrix} 0.2971 & 0.2990 & 0.4039 \\ 0.6392 & 0.2687 & 0.09207 \\ 0.1091 & 0.2264 & 0.6645 \end{pmatrix}$$

# Transition probability matrix for Nigerian Current Account (NCA) from 2014-2022

$$NCA - NET_{2014-2022}(P) = \begin{pmatrix} 0.3637 & 0.3471 & 0.2891 \\ 0.4262 & 0.4667 & 0.1071 \\ -1.7125 & 1.7923 & 0.9203 \end{pmatrix}$$

In order to take on the overall nature of the larger market on Nigerian Current Account Net, we therefore introducing percentage changes of p = .2, .4, .6 to each independent transition probability matrix to see the effect of changes in (NCA) gives the following:

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$$NCA - NET_{2004-2012}(P) = (.2,.4,.4) \begin{pmatrix} 0.3414 & 0.2712 & 0.3874 \\ 0.7216 & 0.2028 & 0.07557 \\ 0.4072 & 0.1307 & 0.4621 \end{pmatrix} = (0.6012 \ 0.2138 \ 0.3850)$$

Taking transpose of the row vector of on the effect of NCA changes gives:

$$NCA - NET_{2004-2012}(P)' = \begin{pmatrix} 0.6012\\ 0.2138\\ 0.3850 \end{pmatrix}$$

$$NCA - NET_{2005-2013}(P) = (.2, .4, .6) \begin{pmatrix} 0.2971 & 0.2990 & 0.4039 \\ 0.6392 & 0.2687 & 0.09207 \\ 0.1091 & 0.2264 & 0.6645 \end{pmatrix} = (0.3806 \ 0.3031 \ 0.5163)$$

Taking transpose of the row vector of on the effect of NCA changes gives:

$$NCA - NET_{2005-2013}(P)' = \begin{pmatrix} 0.3806\\ 0.3031\\ 0.5163 \end{pmatrix}$$
$$NCA - NET_{2014-2022}(P) = (.2,.4,.6) \begin{pmatrix} 0.3637 & 0.3471 & 0.2891\\ 0.4262 & 0.4667 & 0.1071\\ -1.7125 & 1.7923 & 3.2639 \end{pmatrix} = (-0.7848\ 1.3315\ 2.0590)$$

Taking transpose of the row vector of on the effect of NCA changes gives:

$$NCA - NET_{2014-2022}(P)' = \begin{pmatrix} -0.7848\\ 1.3315\\ 2.0590 \end{pmatrix}$$

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Combining each of the transition probabilities with its column vectors to form system of linear equations and also solving independently gives the following :

#### NCA-NET: 2004-2012

$$\begin{pmatrix} 0.3414 & 0.2712 & 0.3874 \\ 0.7216 & 0.2028 & 0.07557 \\ 0.4072 & 0.1307 & 0.4621 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} 0.6012 \\ 0.2138 \\ 0.3850 \end{pmatrix} = \begin{pmatrix} 0.3414P_1 + 0.2712P_2 + 0.3874P_3 & 0.6012 \\ 0.2138 \\ 0.4072P_1 + 0.2028P_2 + 0.07557P_3 = 0.2138 \\ 0.4072P_1 + 0.1307P_2 + 0.4621P_3 & 0.3850 \end{pmatrix}$$

 $P_1 = -0.25$  ,  $P_2 = 1.72$  and  $P_3 = 0.56$ 

#### NCA-NET: 2005-2013

 $\begin{pmatrix} 0.2971 & 0.2990 & 0.4039 \\ 0.6392 & 0.2687 & 0.09207 \\ 0.1091 & 0.2264 & 0.6645 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} 0.3806 \\ 0.3031 \\ 0.5163 \end{pmatrix} \quad \begin{array}{c} 0.2971P_1 + & 0.2990P_2 + & 0.4039P_3 & 0.3806 \\ 0.6392P_1 + & 0.2687P_2 + & 0.09207P_3 = 0.3031 \\ 0.1091P_1 + & 0.2264P_2 + & 0.6645P_3 & 0.5163 \end{pmatrix}$ 

 $P_1 = 0.47, P_2 = -0.26 \text{ and } P_3 = 0.79$ 

#### NCA-NET: 2014-2022

 $\begin{pmatrix} 0.3637 & 0.3471 & 0.2891 \\ 0.4262 & 0.4667 & 0.1071 \\ -1.7125 & 1.7923 & 3.2639 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} -0.7848 \\ 1.3315 \\ 2.0590 \end{pmatrix} \begin{pmatrix} 0.3637P_1 + & 0.3471P_2 + & 0.2891P_3 & -0.7848 \\ 0.4262P_1 + & 0.4667P_2 + & 0.1071P_3 = 1.3315 \\ -1.7125P_1 + & 1.7923P_2 + & 3.2639P_3 & 2.0590 \end{pmatrix}$ 

 $P_1 = -5.30, P_2 = 9.37 \text{ and } P_3 = -7.29$ 

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Table 4: The Summary of Transition Probabilities and additional Statistical Disparities

Year	Dimension	$P_{-redu}$	$P{incr}$	$P{No-ch}$	Mean	STD	Skewness	Kurtosis
2004-	9	0.3414	0.2712	0.3874	0.3333	0.0585	-0.2484	1.5000
2012								
		0.7216	0.2028	0.07557	0.3333	0.3422	0.5988	1.5000
		0.4072	0.1307	0.4621	0.3333	0.1776	-0.6319	1.5000
2005-	9	0.2971	0.2990	0.4039	0.3333	0.0611	-0.6319	1.5000
2013								
		0.6392	0.2687	0.09207	0.3333	0.2792	0.4024	1.5000
		0.1091	0.2264	0.6645	0.3333	0.2927	0.5815	1.5000
2014-	9	0.3637	0.3471	0.2891	0.3333	0.0392	-0.5669	1.5000
2022								
		0.4262	0.4667	0.1071	0.3333	0.1970	-0.6736	1.5000
		-	1.7923	3.2639	1.1146	2.5565	-0.4528	1.5000
		1.7125						

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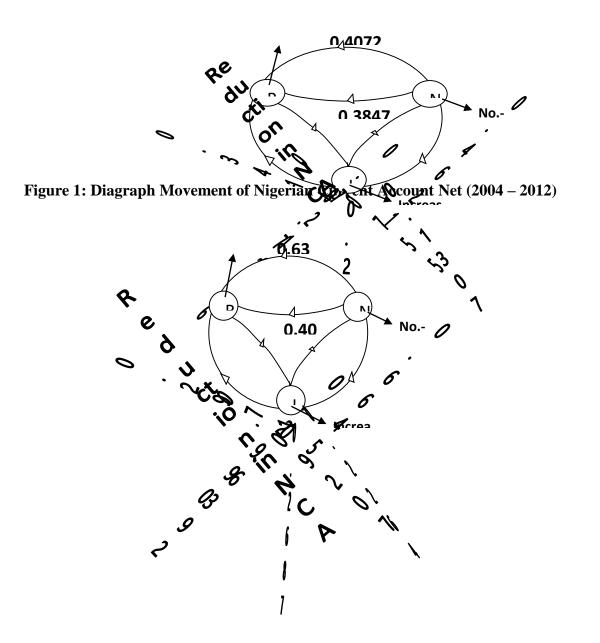


Figure 2: Diagraph Movement of Nigerian Current Account Net (2005 – 2013).

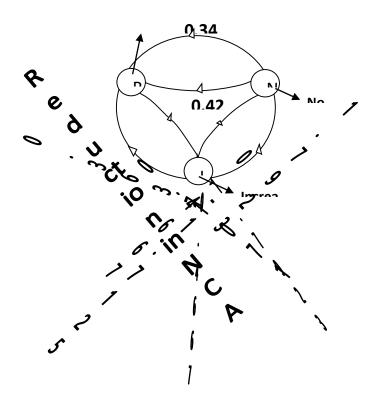
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# Figure 3 : Diagraph Movement of Nigerian Current Account Net (2014 – 2022)

From the stochastic analysis of the transition probability matrices of each independent year of NCA tells about predicting from one state to other as follows:

**NCA** (2004-2012): predicts the probability of imports and exports payments to reducing by 34% ; 27% chance of increasing its imports and exports payments in the near future; 20% chance of no change in imports and exports payments. Also in the same circumstances 72% chance of reducing imports and exports payments; 20% chance of increasing imports and exports payments and 7% chance of no change in imports and exports payments.Finally 41% chance of reducing its imports and exports payments; 13% chance of increasing its imports and exports payments and 47% Chance of no change in imports and exports payments

In NCA (2005-2013): shows the probability of imports and exports payments reducing by 30%; 30% chance of increasing its imports and exports payments in the near future; 40% chance of no change in imports and exports payments. Similarly in the same situations 64% chance of reducing

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imports and exports payments; 21% chance of increasing imports and exports payments and 9% chance of no change in imports and exports payments.

To conclude 11% chance of reducing its imports and exports payments; 23% chance of increasing its imports and exports payments and 66% Chance of no change in imports and exports payments From the results of **NCA** (**2014-2022**): describes the probability of imports and exports payments reducing by 36% ; 35% chance of increasing its imports and exports payments in the near future; 29% chance of no change in imports and exports payments . Equally in the same situations 43% chance of reducing imports and exports payments; 47% chance of increasing imports and exports payments .

In all, -1.7% chance of reducing its imports and exports payments; 1.8% chance of increasing its imports and exports payments and 3.3% Chance of no change in imports and exports payments The above assessments tells the level of surplus made in imports and exports of goods and services, payments made to foreign investors and transfers such as foreign aid. The -1.7% indicates shortages in the goods and services in NCA. Hence, the predicted results provide an eye opener of this stochastic analysis that will enhance their investment decisions The summary of the column vectors which is the changes in Nigerian Current Account (NCA) with their various years stipulates payments changes after three years intervals for both short and long term business plans. In Table 4, shows the comparisons of transition matrices of different years which as follows:

In the years 2004-2012 has the maximum probability of reducing in payments by 72%, the year 2005-2013 has the maximum probability of reducing by 66% and finally in 2014-2022 has the maximum probability of no-change in payments of goods and services by 3.3%. More so, the skewness and kurtosis seen, indicate the distributions were skewed to the right. This remark has financial implications which imply that the investments are profit maximizing. While the kurtosis of the asset values was observed to be describing that the distribution is more heavily-tailed in comparison to the normal distribution. This description informs investors more reliable and effective ways of decision making in terms of imports and exports of goods and services. However, the results of system of equations predicts in both long and short term investments plans for four months in each independent year and covers 2004-2022:

In the quarters of **2004-2012** indicates that: -25% deficits; which means that Nigerians should imports more than it exports. Then in another quarter, Nigerians will experience 172% surplus and should export more. Finally, in the last quarter, the surplus will reduce to 56% but Nigerians should still exports more than it imports.

From **2005-2013**. In the first quarter Nigerians will experience 47% surplus and should export more. The second quarter specifies -26% deficit ; which means that Nigerians should imports more than it exports. Finally, in the last quarter the surplus will increase to 79% and Nigerians should still exports more than it imports.

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In the quarters of **2014-2022** indicates that Nigerian deficit will increase by -520%; which means that Nigerians should imports more than it exports. Then in another quarter Nigerians will experience 937% surplus and should export more. Finally, in the last quarter the deficit will increase to -729% but Nigerians should still imports more than it exports.

More so, Figures 1-3 shows the state transition diagraph of NCA for each year, it suggests that each states communicates well hence is a Markov chain the past and future are independent when the present is known.

# CONCLUSION

The suggestion of Markov chain model and system of equations are precise mathematical tools for exploring the Nigerian Current Account formation; since each finite state communicates for suitable decisions making. Consequently this paper studied stochastic analysis of Markov chain on NCA data (2004-2022). The NCA data were transformed into 3-steps transition probability matrix solution to cover the number of independent years. From the transition matrix of stochastic analysis showed that in 2004-2012 has the highest probability of reducing in payments by 72%, the year 2005-2013 has the highest probability of reducing by 66% and finally in 2014-2022 has the highest probability of no-change in payments of goods and services by 3.3%.

The future NCA data changes were known by introducing the concepts of percentages in each interval of years as column vectors where system of linear equations were developed and solutions were obtained to guide Nigerian economy on various levels of imports and exports of goods and services. Lastly, other statistical disparities were also considered and well discussed in accordingly. These results are informative to Nigerian economy to effectively take vital decisions in both short and long term investment plans.

Nevertheless, introducing delay concept in studying of NCA payment movement will be a good area to explore.

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