The Influence of Markov Chain and Properties of Principal Component Solutions in The Analysis of Share Price Movements for Stock Market

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ABSTRACT: *The stock market performance and operation has been widely recognized as a significantly viable investment field in financial markets. Therefore, this paper studied stochastic analysis of Markov chain and PCA in the closing share price data of Access and Fidelity banks (2016-2022) through Nigeria Stock Exchange. The share prices were transformed into 3-steps transition probability matrix solution to cover this number of years. The striking focus of this is on two merged banks where their transition probability matrix was considered. The future share prices changes were known. The criteria of obtaining four share prices which formed from the two merged banks 2x2 matrices were given and analytical solution of principal component were considered for future stock price changes. From the solution matrix of two merged banks showed that the has the best probability of price increasing in the near future: 12%, best probability of reducing in future by 22% and best probability of no-change in the near future by 21% which is a tool for proper decision making in the day-to-day management of the bank; which shows it is profit making organization and are hopeful for future investment plans both short or long term respectively.*

KEYWORDS: Stochastic Analysis, Markov Chain, Capitalization, Transition Matrix

INTRODUCTION

The vital decisions investors or owners of corporations will undertake; is how to properly allocate funds at every unit of their business. This is due to unpredictability in stock market prices. Therefore, the above constraints need a viable mathematical model that guides investors in decision making. So, an investment, decision making is one of the major gears that affect investors either positively or negatively respectively. When good decisions are taken, the financial strong point of the investments will increase but when incorrect decisions are taken, the businesses begin to smash down.

Though, lots of researchers has written widely on the demonstrating stock price using Markov chain and results gotten in numerous ways; just as in the work of [1] examined stock market prices due to its fluctuations and influences in financial lives and economic health of a country. Their findings showed that stock price is a random work and no investor can alter the fairness and unfairness of a stock price as defined by expectation. [2] Studied Markov process of stock market performance. In their result oil india exhibits a higher chance of being stable with no significant increase or decrease. [3] Studied Markov chain model on share price movement. Result showed GT Bank shares change hands more than the FB Bank. [4] Considered the Markov chain model on forecasting of stocks prices in Nigeria. In their result Markov chain model was determined based on probability transition matrix. [5] Explored the stochastic analyses of Markov chain in finite states. Their work replicated the use of 3-states transition probability matrix which enables them to proffer precise condition of obtaining expected mean rate of return of each stock. ,[6] considered stochastic analysis of share prices. Results showed precise condition of determining expected mean return time for stock price; improving investment decision based on highest transition probabilities. In the same manner, [7] studied the behavior of stock market using Markov chain. The study reveals that regardless of bank's current share price steady state probabilities of share price remain the same all through the iteration. [8] Introduced a Markov chain model for stock market trend forecasting. The study revealed the Markov chain model was more effective to analyze and predict the stock market index and closing stock price under the market mechanism. [9] Studied long run prospects of security prices in Nigeria where the data were collected from the randomly selected banks from the banking sector of Nigeria. The analysis suggested that the price level of Nigerian bank were likely to remain relatively stable in the long run irrespective of the current situations.[10] examined the long run behavior of the closing price of shares of eight Nigerian banks using Markov chain model. They computed limiting distribution transition probability matrix of the of share price and found that despite of the current situation in the market there is hope for Nigerian bank stocks. It was concluded that the results derive from the study will be useful to investors. See for considerable extensions in this area of study [11-17] etc.

The prediction of possible states of share price is more complicated due to the inherent stochastic behavior of prices which rises up and down making lives uneasy. Therefore the share price data of Access and Fidelity banks were used to effectively understand its price movements and their future merging. The above concepts needs stochastic analysis of Markov chain to realistically follow-up the movement of share prices that is characterize by volatility. The model presented in this study will give Fidelity and Access clear directions when the two banks merged in terms of decision making concerning share prices, as it helps them understand the dynamics and patterns of the share prices that would be followed for optimum trading and profit. Secondly it will account the level of proportion of first PC of future price changes.

The aim of this paper is to study the effect of Markov chain in finite states with Principal Component Analysis (PCA) in the analysis of share prices of two merged banks. This paper extends the work of [18] by incorporating PCA to study the variations in the two merged banks.

This paper is arranged as follows: Section 2.1 presents the materials and method, Results and Discussion presented in Section 3.1 and the paper is concluded in section 4.1.

MATERIAL AND METHODS

The purpose of this paper on Markov chain we start from defining stochastic process. It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 1: A stochastic process X is said to be a Markov chain if Markov property is satisfied :

$$P\left(X_{n+1} = j \, / \, X_0, X_1, \cdots, X_n\right) = P\left(X_{n+1} = j \, / \, X_n\right) \tag{1}$$

For all $n \geq 0$ and $i, j \in S \, (\text{state space})$.

It is sufficient to know that the Markov property given (1) is equivalent to easy of the following for each $j \in S$.

$$P\left(X_{n+1} = j \, / \, X_{n1}, X_{n2}, \cdots, X_{nk}\right) = P\left(X_{n+1} = j \, / \, X_{nk}\right) \tag{2}$$

$\left(\text{for any } n_1 < n_2 <, \cdots, n_k \leq n\right)$

Assuming $X_n = i$ implies that the chain is in the *ith* state at the *nth* step.it can also be said that the chain' having the value i' or ' being in state i'. The idea behind the chain is described by its transition probabilities:

$$P\left(X_{n+1} = j \, / \, X_n = i\right) \tag{3}$$

They are dependent on i, j and n.

Definition 2: The chain X is said to be homogeneous if the following are stated below

$$P\left(X_{n+1} = j \, / \, X_n = i\right) = P\left(X_1 = j \, / \, X_0 = i\right) \tag{4}$$

For all $n, i, j.$

The transition matrix $\mathrm{P} = \left(P_{ij} \right)$ is $n \times n$ matrix of transition probabilities.

$$P_{ij} = P\left(X_{n+1} = j \,/\, X_n = i \right) \tag{5}$$

Hence, the transition probabilities with homogenous Markov chain are always stationary at a point.

Theorem 3: Suppose P is a stochastic matrix which implies the following:

i) P has non-negative entries or $\mathrm{P}_{ij} \geq 0$ (ii)

$$\sum_j P_{ij} = \sum_j P\left(X_{n+1} = j \,/\, X_n = i \right) = \sum_j P\left(X_1 = j \,/\, X_0 = i \right)$$

which is stationarity or point of convergence.

Proof:(i) each associated entry in P is a transition probability P_{ij} and being probability $P_{ij} \geq 0$.

(ii) $\sum_j P_{ij} = \sum_j P\left(X_{n+1} = j \,/\, X_n = i \right) = \sum_j P\left(X_1 = j \,/\, X_0 = i \right)$

Which is stationarity.

$$P\left(X_i \in S \,/\, X_0 = i \right) = 1.$$

Theorem 4 :(Chapman-Kolmogorov Equations).

$P_{ij(m+n)} = \sum_{r=i}^{n} P_{ir(m)} \, P_{rj(n)}$ Since $\mathrm{P}_{m+n} = \mathrm{P}_m \mathrm{P}_n$ and so on $\mathrm{P}_n = \mathrm{P}^n$ the *nth* power of P .

$$P_{ij(m+n)} = P\left(X_{m+n} = j \,/\, X_0 = i \right)$$

Proof:
$$\sum_r P\left(X_{m+n} = j, X_m = r \,/\, X_0 = i \right)$$

$$\sum_r P\left(X_{m+n} = j \,/\, X_m = i \,/\, X_0 = i \right) P\left(X_m = r \,/\, X_0 = i \right)$$

Using the following in probability rule:

$P\left(A \cap B \,/\, C \right) = P\left(A \,/\, B \cap C \right) P\left(B \,/\, C \right)$ and setting

$A = \left\{ X_{m+n} = j \right\}, \mathrm{B} = \left\{ X_m = r \right\}, \text{and } \mathrm{C} = \left\{ X_0 = i \right\}$

Using Markov property yields

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$$P_{ij(m+n)} = \sum_r P\left(X_{m+n} = j \,/\, X_m = r\right)P\left(X_m = r \,/\, X_0 = i\right)$$

$$\sum_r P_{rj(n)} P_{ir(m)}$$

$$\sum_r P_{1r(m)} P_{r1(n)}$$

Hence $P_{m+n} = P_m P_n$ and so $P_n = P^n$, the power *of* P.

To obtain an estimates of the transition probability as follows

$$P_{ij} = P\left(X_t = j \,/\, X_{t-1} = i\right), \; for \; j = 0,1,2,3,\cdots,N$$

$$P_{ij} = \begin{cases} P \; \text{if} \; j = 1 + j \\ q = 1 - P \; \text{if} \; j = i - j \\ 0 \; \text{otherwise} \end{cases}$$

where $k + 1$ is the number of states.

$$\left. \begin{array}{l} n_{ij} = \displaystyle\sum_{i=1}^{n} P_{ij} \; \text{for i } j{=}0,2,3 \\[2mm] \dfrac{n_{ij}}{n_i} \; \text{for } i \; j = 0,1,\cdots k \end{array} \right\} \tag{6}$$

However, for $k = 3$ is an estimate of the transition matrix.

$$\hat{P}ij(\text{FIDELITY})_{2016-2022} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix} \tag{7}$$

$$\hat{P}ij(\text{ACCESS})_{2016-2022} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix} \tag{8}$$

$$\hat{P}ij(\text{ACCESS}- \text{FIDELITY})_{2016-2022} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix} \tag{9}$$

Setting i $j = 0,1,2$ for $k = 3$

Developing Markov Chain Model of Access Fidelity banks Share Prices

For proper accuracy of Markov chain model for future events; it needs to be developed for prediction of share price movement. The initial share prices needs to be in three finite states as follows:

R: represents the probability of share price reducing in near future, **I** : represents the probability of share price increasing in near future, **NO-change** : represents the probability of share price not changing in near future

However, probability of transition matrix shows the proper explanation of Markov chain. Every element in the matrix communicates. In order to form three states of Markov process we need to have the following table below:

Table 1: Transition Probability Matrix

State	1	2	3	Total of Row
1	P_{11}	P_{12}	P_{13}	T_1
2	P_{21}	P_{22}	P_{23}	T_2
3	P_{31}	P_{32}	P_{33}	T_3

In each entry P_{ij} indicates the number of times a transition is made from one state i to state j .

the transition matrix is computed by simply dividing every element in each row through the total of each row. Nevertheless, this project studies Fidelity share price data collected from [18].

Principal component Analysis of the share price variables

Definition 3: According to [19] suppose $\underline{X}$ has a joint distribution which has a variance matrix $\sum$ with eigenvalues $\lambda_1, \geq \lambda_2 \geq ... \geq \lambda_p \geq 0.$ consider the random variables $y_1...y_p$ which are linear combination of the $X_1's$ ie:

$$\left. \begin{array}{c} y_1 = \underline{l}'\underline{X} = l_{11}X_1 + ... + l_{p_1}\lambda_p \\ \vdots \\ y_p = \underline{l}'\underline{X} = l_1 pX_1 + ... + l_{p_1}\lambda_p \end{array} \right\}$$ (10

The $y_i's$ will be PC if they are uncorrelated and the variances of $y_{1,}y_2$ are as large as possible.

Recall that if $y_i = l_i'X$. In order to look at the amount of information that is in y_1 .We can consider

the proportion of the total population variance due to y_i

$$\frac{\lambda_i}{\lambda_1 + \lambda_2 + \cdots + \lambda_p}, i = 1,..., p$$ (11)

Hopefully the proportion is large for eg=1,2, 3.

 RESULTS AND DISCUSSION

The data for this paper is gotten from the work of [18].To demonstrate the closing share market price performances of two banks merged in finite states . The share price covers from 2016-2022 retrievable from Nigeria Stock Exchange (NSE).

Table 1: Share price of Fidelity Bank, PLC from 2016-2022

Share price movements	Reducing(R) its share Price	Increasing(I) its share price	No(N) change in price	Row totals
R	415	62	138	615
I	61	121	81	263
N	139	80	384	603

Table 2: Share price of Access Bank, PLC from 2016-2022

Share price movements	Reducing(R) its share Price	Increasing(I) its share price	No(N) change in price	Row totals
R	410	80	126	616
I	79	98	92	269
N	127	91	378	596

Table 3: Share price of two Banks merged, from 2016-2022

Share price movements	Reducing(R) its share Price	Increasing(I) its share price	No(N) change in price	Row totals
R	825	142	264	1231
I	140	219	173	532
N	266	171	762	1199

Transition probability matrix for Access Fidelity banks merged according to [18].

$$ACCESS - FIDELITY = \begin{pmatrix} 0.6702 & 0.1154 & 0.2145 \\ 0.2632 & 0.4117 & 0.3252 \\ 0.2219 & 0.1426 & 0.6355 \end{pmatrix}$$

Minimum share price criteria: 2×2 matrix[5].

$$ACCESS - FIDELITY = \begin{pmatrix} 0.1154 & 0.2145 \\ 0.2219 & 0.1426 \end{pmatrix}$$

we formed the matrix from the estimates of probability transition matrix of two banks merged which information of the share price movements will account the total proportion variability in the share price.

Table 4: Transition Probability Matrix of two merged Banks Share Market Prices with Means, Standard deviations, Kurtosis and Skewness

a_{11}	a_{12}	a_{13}	Mean	Std	Kurtosis	Skewness
0.6702	0.1154	0.2145	0.3334	0.2959	1.500	0.6189
0.2632	0.4117	0.3252	0.3334	0.0746	1.500	0.1987
0.2219	0.1426	0.6355	0.3333	0.2647	1.500	0.6364

In Table 4, the mean indicates the probability of share price changes on average of Access bank which shows 0.3333 throughout the period of investments, see column 4. The standard deviations in column 5 indicate levels of different price changes which is always affected by volatility. Then kurtosis measures the tailedness of entire share prices. Finally , the share price skewness measures level of distortion in the data set which guides an investor on basis of decision making.

Table 5: Variations of future share prices of two merged banks according to the trading days

Trading days	0.1154	0.2145	0.2219	0.1426
2	0.2308	0.429	0.4438	0.2852
4	0.4616	0.858	0.8876	0.5704
6	0.6924	1.287	1.3314	0.8556
8	0.9232	1.716	1.7752	1.1408
10	1.154	2.145	2.219	1.426
12	1.3848	2.574	2.6628	1.7112
14	1.6156	3.003	3.1066	1.9964
16	1.8464	3.432	3.5504	2.2816
18	2.0772	3.861	3.9942	2.5668
20	2.308	4.29	4.438	2.852

In Figure 3, a little increase in the number of trading days also increases future share prices of two merged banks in its operations. It also implies that profit increases over time; as gives business cycles to be more flexible adapting to market demands. ; the benefits of this assessment is to avert severe depletion of capital investments which may endanger profit making throughout the trading period of the capital investments.

However , the above valuations of the Fidelity and Access banks offers an eye opener of these stochastic analysis that will enhance investment decisions, The entire entry stipulates price changes for short and long term business plans.

Principle Component Analysis of two banks merged share price Movement Variations

$$ACCESS - FIDELITY = \begin{pmatrix} 0.1154 & 0.2145 \\ 0.2219 & 0.1426 \end{pmatrix}, \; ACCESS - FIDELITY - \lambda I = 0$$

Solving the above share price matrix gives:

$$\lambda_1 = -0.0896, \; \lambda_2 = 0.3476$$

Solving for $\lambda_1 = -0.0896$, we have the following systems of equation

$$0.205K_1 + 0.2145K_2 = 0 \tag{4.1}$$

$$0.2219K_1 + 0.2322K_2 = 0 \tag{4.2}$$

From (4.1) $0.205K_1 = -0.2145K_2$, $K_2 = \dfrac{0.205}{0.2145} = -0.9557$ putting K_2 in (4.2) gives

$$0.2219K_1 + 0.2322(-0.9557) = 0, \; 0.2219K_1 = 0.22191354 = K_1 = \dfrac{0.22191354}{0.2219} = 1.0001$$

Any vector of the form say form:

$$K_1 = \begin{pmatrix} 1.0001 \\ -0.9557 \end{pmatrix} = \begin{pmatrix} 1.0001C \\ -0.9557C \end{pmatrix}; \; say \text{ is an eigenvector corresponding to } \lambda_1 = -0.0896$$

$$-0.2322K_1 + 0.2145K_2 = 0 \tag{4.3}$$

$$0.2219K_1 - 0.205K_2 = 0 \tag{4.4}$$

From(4.3) $-0.2322K_1 = -0.2244K_2$, $K_2 = \dfrac{0.2322}{0.2145} = 1.0825$ putting K_2 in (4.4) gives

$$0.2219K_1 - 0.205(1.0825) = 0, \; 0.2219K_1 = 0.2219125 = 0_1, \; K_1 = \dfrac{0.2219125}{0.2219} = 1.0001$$

Any vector of the form say form:

$$K_2 = \begin{pmatrix} 1.0001 \\ 1.0825 \end{pmatrix} = \begin{pmatrix} 1.0001C \\ 1.0825C \end{pmatrix}; \; say \text{ is an eigenvector corresponding to } \lambda_2 = 0.3476$$

To obtain anormalised eigenvectors for share price of Access bank:

$$K_1'K_1 = 1, \ (1.0001C \ -0.9557C)\begin{pmatrix} 1.0001C \\ -0.9557C \end{pmatrix}, \ 1.00020001C^2 + 0.912025C^2 = 1 \ , 1.91222501C^2 = 1$$

$$C^2 = \frac{1}{1.91222501}, \ C = \frac{1}{\sqrt{1.91222501}}, \ e_1 = \begin{pmatrix} \dfrac{1.0001}{\sqrt{1.91222501}} \\ \dfrac{-0.9557}{\sqrt{1.91222501}} \end{pmatrix}, \begin{pmatrix} 0.7232 \\ -0.6911 \end{pmatrix}$$

$$K_2'K_2 = 1, \ (1.0001C \ \ 1.0825C)\begin{pmatrix} 1.0001C \\ 1.08250C \end{pmatrix}, \ 1.00020001C^2 + 1.17180625C^2 = 1 \ , 2.17200626C^2 = 1$$

$$C^2 = \frac{1}{2.17200626}, \ C = \frac{1}{\sqrt{2.17200626}}, \ e_1 = \begin{pmatrix} \dfrac{-1.0001}{\sqrt{2.17200626}} \\ \dfrac{1.0825}{\sqrt{2.17200626}} \end{pmatrix}, \begin{pmatrix} -0.7232 \\ 0.7345 \end{pmatrix}$$

$Y_1 = e_1'K = 0.7232K_1 - 0.6911K_2$

$Y_2 = e_2'K = 0.7345K_1 + 0.7345K_2$

To calculate the principal component of Access bank share price accounted for the ist PC

$$\lambda_1 = -0.0896, \ \lambda_2 = 0.3476 \ , \frac{\lambda_1}{\lambda_1 + \lambda_2} = 0.5 = 50\%$$

Two eigenvalues represent the total amount of Fidelity share price variance that can be explained by the principal component. The $\lambda_1 = -0.0896$ represents the levels of losses made all through the trading days by Fidelity bank PLC. So $\lambda_2 = 0.3476$ is greater than zero which is a good sign of high level of investment return in the side of the Fidelity bank whose aim and passion is to maximize profit. More so, the eigenvectors determine the direction of the share price in terms of changes in short-run and long-run respectively. The 50% shows the average returns of two banks merged in the future.

The two banks merged: Access and Fidelity (2017-2021): Has 67% of reducing its price; 12% chance of increasing its price in the near future; 21% chance of no change in price. Also in the same circumstances 26% chance of reducing its price; 41% chance of increasing its price and 33% chance of no change in price.

Finally 22% chance of reducing its price 14% chance of increasing its price and 64% Chance of no change in price. In all, the overall predicted prices for the above companies gives: 22% chance of reducing its price,14% chance of increasing its price and 64% chance of no change in price

CONCLUSION

Markov chain is a mathematical tool used in modeling share price movement of stochastic processes; since each finite state communicates for proper management decision making. Therefore this project studied stochastic analysis of Markov chain and PCA in the closing share price data of Access (2016-2022) via Nigeria Stock Exchange. The share prices were transformed into 3-steps transition probability matrix solution to cover this number of years. The future share prices changes were known. The criteria of obtaining four share prices which formed 2x2 matrix were given from two merged banks and analytical solution of principal component were considered for future stock price changes. From the solution matrix of stochastic analysis showed that **Access bank, PLC** has the best probability of price increasing in the near future: 12%, best probability of reducing in future by 21% and best probability of no-change in the near future by 20% which is a tool for proper decision making in the day-to-day management of the bank. More so, From the **Fidelity bank, PLC** has the best probability of price increasing in the near future: 10%, best probability of reducing in future by 23% and best probability of no-change in the near future by 22% which is a tool for proper decision making in the day-to-day management of the bank. Finally from the two banks merged; has the best probability of price increasing in the near future: 12%, best probability of reducing in future by 67% and best probability of no-change in the near future by 21% which is a tool for proper decision making in the day-to-day management of the bank. In the analysis of PC 50% shows the average returns of two banks merged in the near future. However, this study investigates three states case of transition matrix with PCA, the stochastic differential equation problem is suggested as an interesting area of further study.

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