British Journal of Multidisciplinary and Advanced Studies:
Engineering and Technology, 4(3),31-46, 2023
Print ISSN: 2517-276X
Online ISSN: 2517-2778
https://bjmas.org/index.php/bjmas/index
Published by the European Centre for Research Training and Development UK

Numerical Approximation of Oscillatory Initial Value Problem Using the Homotopy Analysis Algorithm

Obafaiye P.O, Imoni S.O, Lanlege D. I.
Department of Mathematics, Federal University Lokoja, Kogi State, Nigeria
paulineolayemi518@gmail.com

doi: https://doi.org/10.37745/bjmas.2022.0206 Published May 29, 2023

Citation: Obafaiye P.O, Imoni S.O, Lanlege D. I. (2023) Numerical Approximation of Oscillatory Initial Value Problem Using the Homotopy Analysis Algorithm, *British Journal of Multidisciplinary and Advanced Studies*: Engineering and Technology, 4(3),31-46

ABSTRACT: *In this paper, we present numerical approximation for oscillatory initial value problems (IVPs) using the homotopy analysis algorithm. The convergence of the method is discussed and numerical experiments are presented to illustrate the computational effectiveness of the algorithm. The results obtained are in good agreement with the exact solutions and Adomian decomposition method (ADM). These results show that the algorithm introduced here is accurate and easy to apply without linearization.*

KEYWORDS: numerical analysis, initial value problems, differential equations, homotopic analysis

INTRODUCTION

Oscillatory initial value problems arise in mathematical models for problems in mechanics, physics and engineering. These are more difficult to solve analytically, hence it seems more natural to provide direct numerical methods for solving such initial value problems, it is the purpose of this paper to discuss the homotopy analysis algorithm for approximation of oscillatory IVPs. Homotopy Analysis Algorithm (HAA) is a semi-analytic technique used to solve non-linear ordinary differential equations. The Homotopy Analysis Algorithm employs the concept of homotopy in topology to generate a convergent series solution of non-linear systems. This is enabled using a Homotopy-Taylors series to deal with the non-linearity in the system. The Homotopy Analysis Method was proposed by Liao 1992 and the method provides a convenient way to control and adjust the convergence region and rate of series approximation. Ghoreishi et al. (2011) applied HAA to solve a model for HIV infection of CD4+ T- cells. Fallahzadeh and Shakibi (2015) applied homotopy analysis algorithm (HAA) to solve Convection-Diffusion equation. Mkharrib and Salem (2021) studied new algorithm of the optimal homotopy asymptotic method for solving Lane-Emden equations. Omar (2021a) carried out homotopy analysis-based hybrid genetic algorithm and secant method to solve IVP and higher-order BVP. This work will extend the homotopy analysis algorithm to approximate the solution of oscillatory initial value problems.

Homotopy Analysis Algorithm

Consider the non-linear differential equation

$$N[U(X)]=0 \qquad (2.1)$$

N is a non-linear differential operator, X denotes the independent variable $U(X)$ is an unknown function. According to Liao (1992) , the zero order deformation equation is given as

$$[1-q]L(x;q)-Uo(x)]qhN[\Phi(x;q) \qquad (2.2)$$

Where $q\in(0;1)$ is an embedding parameter, $h\neq 0$, L is an auxiliary linear operator U_0 is the initial guess of $U(X)$ and $\phi(x;q)$ is an unknown function. In this work we assume $h=-1$ when $q=1$ in equation (2.2) and $\phi(x;q)=U_0(x)$

$\phi(x;1)=U(x)$

Since $\phi(x;q)$ depends on the parameter q, expanding $\phi(x;q)$ by Taylor's series with respect to q

$$\phi(x;q)=\phi(x;0)+\sum_{n=2}^{\alpha}Un(X)q \quad \phi(x;q)=Uo+\sum_{n=1}^{\infty}Un(X)q^{n} \qquad (2.3)$$

$$Un=\frac{1}{n!}\frac{\partial^{n}\phi(x;q)}{\partial q^{n}}|q=0 \qquad (2.4)$$

From equation (2.2) when $q=1, we\ get\ \phi(x;1)Uo+\sum_{n=1}^{\infty}Un(0)$ $u_n(x)$ can be deduced using the zero-order deformation equation (2.2). Differentiating (2.2) n times with respect to the embedding parameter q and q=1, divide by n! And get the nth order deformation equation

$$L[U_n(X)-X_nU_{n-1}9X)]=hD_{n-1}[N(\phi(x;q)] \qquad (2.5)$$

Where

$$D_{n-1}[N(\phi(x;q)]=\frac{1}{(n-1)!}\frac{\delta^{n-1}[N(x;q)]}{\delta q^{n-1}}$$

For example, consider the non-linear differential equation $\frac{\partial u}{\partial x} - 2u^2 = 0$ with initial condition $Uo(0) = 1$.

$$L[\phi(x)] = \phi^1(x)$$

$$N[\phi(X)] = \phi^1(X) - 2\phi(X)^2 \quad (2.6)$$

Let $Uo(0) = 1,\ u_0^1(x) = 0$

From equation (2.3),we obtain $u_o = 0$ and $u_n(0) = 0 for\ all\ n \geq 1$Let

$$\phi = \phi_o(x) + \sum_{n=1}^{\infty} \phi_n q^n = \phi_o + \phi q + \phi_2 q^2 + \phi_3 q^2 + ...$$

$$= \phi_o(x) + \sum_{n=1}^{\infty} \phi_n q^n = \phi^1{}_o + \phi_1^1 q + \phi^1{}_2 q^2 + \phi^1{}_3 q^2 + ... \quad (2.7)$$

using (2.1),we obtain

$$1[1-q] = [L[\phi(X) - \phi_N(X)] = hq[N[\phi(X)]\ (1-q)[\phi_1 - \phi_0^1] - 2\phi^2]$$

Substituting (2.5), (2.6) and (2.7) and get

$$= \phi_2 + \phi_1^1 q + \phi_2 q^2 + \phi_3 q^2 + ...) - \phi_0^1 \_ \beta[\phi_0 + \phi^1 q + \phi_2^1 q + \phi_3 q^3 + ... \quad (2.8)$$

Differentiating (2.8) with respect to q and get the first derivative, to obtain.

$$(1-q)[\phi_1^1 + 2\phi_2^1 q + 3\phi_3^1 q^2 + ...] + [-1[\phi 1_1^1 + \phi_2^1 q + \phi 1_3 q^3 = hq[\phi_1^1 + 2\phi_2^1 q^3 + 3\phi 1_3 q^2 + ...]$$
$$-4[\phi_0 + \phi_1 q + \phi_2{}^2 q^2 + \phi q^3 + ... (2.9)$$

Differentiating (2.1) with respect to q to get first derivative, and obtain

$$(1-q)[\phi_1^1 + 2\phi_2^1 q + 3\phi_3^1 q^2 + ...] + [-1[\phi 1_1^1 + \phi_2^1 q + \phi 1_3 q^3 = hq[\phi_1^1 + 2\phi_2^1 q^3 + 3\phi 1_3 q^2 + ...]$$
$$-4[\phi_0 + \phi_1 q + \phi_2{}^2 q^2 + \phi q^3 + ... (2.10)$$

$$+\phi_2 + 2\phi_2 q + 3\phi_3 q^2 + ...]h[\phi_1^{10} + \phi_2^1 q^3 + ...] - 2[\phi_0 + \phi_1 q + \phi_2 q^2 + \phi^3 q^3 .... \quad (2.11)$$

When $q = 0$ in (2.11) to get

$$\phi_1^1 = h[\phi_0^1 - 2\phi_0^2]$$

Since$u_0(x) = 1$, hence $\varphi_0(x) = 1$, then $\phi_0^1(x) = 0$,and

$\phi_1^1 = h[0-(L)^2]$

$\phi_1^1 = -2h$

Integrating both sides, to obtain

$\phi_1 = -2h + c$ (2.12)

since $u_n(0) = 0, \forall_n \geq 1$, thus $\varphi_n(1) = 0, \forall_n \geq 1$, the $\varphi_1(0) = 0$, so $c = 0$

Thus $\phi_1 = -2hx$ Which is the first derivative of HAA To get the second derivative for (2.12) differentiate the first derivative one time to get

$$3\phi_3^1 q^1 + ...) + (-1)[\phi_1^1 + 2\phi_2^1 q + 3\phi_3^1 q^2 + ...)] + hq[2\phi_2^1 + 6\phi_3^1 q...] - 4[\phi_0 + \phi_1 q + \phi_2 q^2 + \phi_3 q^3 ....]$$

$$-(2\phi_2 + 6\phi_3 q + ...\} - 4[\phi_1 + 2\phi_2 q + 3\phi_3 q^2 + ...)[\phi_1 + 2\phi_2 q + 3\phi_3 q^2 +$$
$$h[\phi_1^1 + 2\phi_2^1 q + 3\phi_3^1 q + ...] - 4[\phi_0 + \phi_1 q + \phi_2 q^2 + \phi_3 q^3 + h[\phi_0 + 2\phi_2^1 q + 3\phi_3^1 q^2 + .$$

$$-4[\phi_0 + \phi_1 q + \phi_2 q^2 + \phi_3 q^3 + ..] + [\phi_{12} + 2\phi_3 q^2 + 3\phi_3 q^2 + ...] \quad (2.12)$$

Let $q = 0$ in (2.12), to get

$2\phi_2^1 - \phi_1^1 - \phi_1^1 = h[\phi_1^1 - 4\phi_0\phi_1]$

But $\phi_0 = 1, \phi_1 = -2h$ and then $\phi_1^1 = -2h + 8h^2 x - 2h$

Integrating both sides, to obtain

$\phi_2 = -2h^2 x + 4^2 x^2 - 2hx + c$ Since $\phi_2(0) = 0$ then $c = 0$,Thus

$\phi_2 = -2h^2 x + 4h^2 x^2 - 2hx$ (2.13)

Which is the second derivative, to get the third derivative for (2.13) differentiate three times to differentiate the second derivative one time.

$$[1-q][6\phi_3^1 + ...] + [2\phi_2^1 + 6\phi_3^1 q + ..](-1)[2\phi_2^1 + 6\phi_2 + 6\phi_3^1 q + ..](-)[2\phi_2^1 + 6\phi_3^1 q + ...$$

$$=hq[(6\phi_3+..)-4[\phi_0+\phi_1 q+\phi_2 q_2+\phi_3 q\ +....][6\phi_3+...]-4[2\phi_2+6\phi_3 q+...][\phi_1+2\phi_2 q+3\phi_3 q^2+..]$$
$$-4[\phi_1+2\phi_2 q+3\phi_3 q^2+\phi_3 q^2+\phi_3 q^3+..][6\phi_3+...]-4[2\phi_2+6\phi_3 q+..][\phi_1+2\phi_2 q+3\phi_3 q^2+...]$$
$$-4[\phi_1+2\phi_2 q+3\phi_3 q^2+][2\phi_2+6\phi_3 q+..]-4[\phi_1+2\phi_2 q+3\phi_3 q^2+...][2\phi_2 q+6\phi_3 q]$$
$$=h[2\phi_2^1+6\phi_3^1 q+..]=4[\phi_0+\phi q+\phi_2 q+\phi_3 q+..]-4[\phi_1+2\phi-2q+3\phi_3 q^2+,,,,]$$
$$\phi[\phi_1+2\phi_2 q+3\phi_3 q^2+...]+h[2\phi_2^1+6\phi_3^1 q+..]-4[\phi_0+\phi_1 q+\phi_2 q^2+\phi^3 q^3+2\phi\ +6\phi_3 q+..]$$
$$4[\phi_1+2\phi_2 q^2+3\phi_3 q^2+\phi_1+2\phi_2 q+3\phi_3 q^2+...]$$
$$+h[2\phi_2^1+6\phi_2^1 q+...]-4[\phi_0+\phi_1 q+\phi_2 q^2+\phi_3 q^3+2\iota_2+6\phi_3^1 q+...]$$

Let $q=0$ in (2.13) . $6\phi_3^1-2\phi_2^1=h[2\phi_2^1-8\phi_0\phi_2-4\phi_1\phi_1]+h[2\phi_2^1-8\phi_0\phi_2-4\phi_1\phi_1]$

$6\phi_3^1=6\phi_2^1=3h[2\phi_2-8\phi_0\phi_2-4\phi_1\phi_1]$,

Hence $\phi_2^1=\frac{1}{2}h[2\phi_2^1-8\phi_0\phi_1=4\phi_1\phi_1]...$
(2.14)

Using ϕ_0-1 , $\phi=-hx$, $\phi_1^1=-2h$

$\phi_2=-2h^2x+4h^2x^2-2h$ and $\phi_1^1=-2h^2+8h^2x2h$

Then $\phi_3^1=(-2h^2+8h^2)(-2h)=\frac{1}{2}h[2(-2h^2x\_2h)-8(1)(-2h^2x+4h^2x^2=2hx)-4(-2hx)^2$ Or

$\phi_3^1=\frac{1}{2}h[-4h^2+16h^2x-4h+16h^2x-32h^2x+16hx-16h^2x^2]-2h^2+8h^2x^2-2h$

Hence, $\phi_3^1=-2h[h^2+2h+1]+16h^2x(h+1)-24h^3x^2$

Integrating both sides, to get. $\phi_3^1=2h[h^2 2h+1]x+8h^2(h+1)-8h^3x^3+c$

The third approximation for $\varphi(x)$ is

$$\phi_3=\phi_0+\sum_{i=0}^{3}\phi_n$$

i.e., $\phi_3^1=1-2hx-2h^2x+4h^2x^2-2hx-2h[h^2+2h+1+8h^2x^2(h+1)-8h^3x^3$
(2.15)

For n^{th} approximation of $\phi(x)$

$$\phi_n = \phi_0 + \phi_1 + \phi_2 + \phi_3 + ...\phi_n$$

Hence, $\phi_0(x) = \phi_0 + \sum_{n=1}^{n} \phi_n = \phi_1 + \phi_2 + \phi_3 + ...\phi_n$

Convergence Analysis

Theorem: Suppose that $u(x)$ is a Banach space with suitable norm || .|| ,say ||.||∞, over which the sequence $\phi_n(x)$ is defined for a prescribed value of h, Assume also that the initial approximation $\phi_0(x)$ remains inside the domain of the solution $u(x)$ Taking r ∈R as a constant, the following statements holds. If there exist some r ∈ [0, 1] such that for all k ∈ N, we have $\|\phi_k + (x)\|^1 r \|\phi_k(x)\|$**,** Then the series solution $u(x) = k(x)qk$ converges absolutely at $q = 1$ over the domain of x. Proof Indeed, this is a special case of Banach fix point theorem. See Gambari (2014).

Numerical Examples

To demonstrate the effectiveness of the algorithm in this study, we consider the following two examples, the HAA computation results are as presented in the table.

Example 4.1 Consider the fourth order oscillatory initial value problem

$$y^{iv} = 5y'' - 4y$$

with the initial conditions

$$y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 1$$

Exact solution: $1 + \frac{1}{6}\sin 2x$

Source: Bataineh (2009)

To solve the equation by Homotopy Analysis Algorithm with the initial approximation

$$y(x) = y\ (0) + y'(0)x + y''(0)x^2 + y'''(0)x^3$$

$$y(x) = 1 + \frac{1}{6}x^3$$

And linear operator

British Journal of Multidisciplinary and Advanced Studies:
Engineering and Technology, 4(3),31-46, 2023
Print ISSN: 2517-276X
Online ISSN: 2517-2778
https://bjmas.org/index.php/bjmas/index
Published by the European Centre for Research Training and Development UK

$$L[\phi(x;q)] = \frac{\delta^2\phi(x;q)}{\delta x^4}$$

With the property

$$L[c_1 + c_2 + c_3 + c_4] = 0$$

Where $c,[c = 1,2,3,4]$ are constants of integration for m ≥ 1, the mth order deformation with initial conditions will be

$$y_m(0) = 0, y'm(0) = 0, y''m(0) = 0, y'''m(0) = 1$$

Where $Rm(y)_{m-1} = y'''_{(m-1)}(x) + 5x''_{(m-1)}(x) + 4y_{(m-1)}(x)$

The solution of the mth order deformation for m ≥ 1 $y_m(x) = y_m y_{m-1}(x) + hL^{-1}R_m(y_{m-1})$

Hence $y'(x) = \frac{1}{6}hx^4 + \frac{1}{24}hx^5 + \frac{1}{1260hx^7}$

$$y^2(x) = \frac{1}{6}hx^3 + \frac{1}{6}hx^2x^4 + \frac{1}{24}h^2x^6 + \frac{1}{1260}hx^7 + \frac{29}{5040}h^2x^9 + \frac{1}{5230}h^2x^9 + \frac{1}{2494800}h^2x^{11}$$

.. Then the series solution expression can be written in the form

$y(x) = y(0) + y'(x) + y''(x) + y - 3(x) + ...$ .... And so forth

, hence the series solution when h = −1 is

$$y_i(x) = -\frac{1}{6}x^4 + \frac{1}{24}x^5 + \frac{1}{1260}x^7$$

$$y_2(x) = \frac{1}{36}x^6 + \frac{5}{1008}x^7 + \frac{1}{2520}x^8 + \frac{1}{9072}x^9 + \frac{1}{2494800}x^{11}$$

$$y_3(x) = \frac{-5}{2010}x^8 + \frac{25}{7257}x^9 - \frac{1}{22680}x^{11} + \frac{1}{133056}x^{12} + \frac{1}{25945920}x^{13} + \frac{1}{2043241200}x^{15}$$

$$y_4(x) = \frac{5}{36288}x^{10} + \frac{25}{1596672}x^{11} + \frac{1}{399168}x^{12} + \frac{15}{1550755}x^{13} + \frac{1}{90810720}x^{14} + \frac{1}{544864320}x^{15} + \frac{1}{8142948000}x^{16} + \frac{1}{2778803250}x^{17} + \frac{1}{4715761734200}x^{19} + ...$$

And so forth.

Hence the series solution is

$$y(x)=1+\frac{1}{6}x^3+\frac{1}{6}x^4+\frac{1}{24}x^5+\frac{1}{36}x^5+\frac{1}{240}x^7-\frac{1}{480}x^8-\frac{17}{2597}x^9+\frac{17}{181144}x^{10}+..$$

This converges to the ADM solution

Approximation of Homotopy Analysis Algorithm

X, 0

$1 + (0)^3 0.1666 = 1$

X, 0.1

$1 + (0.1)^3 0.1666=1.$

X, 0.2

$1 + (0.2)\ ^3 0.1666 = 1.0013$

X.(0.3)

$1 + (0.3)^3 0.1666 = 1.0045$

X,(0.4)

$1 + (0.4)^3 0.1666 = 1.0107$

X, (0.5)

$1 + (0.5)^3 0.1666 = 1.0208$

X, (0.6)

$1 + (0.6)^3 0.1666 = 1.0360$

X, (0.7)

$1 + (0.7)^3 0.1666 = 1.0571$

X, (0.8)

$1 + (0.8)^3 0.1666 = 1.0571$

X, (0.9)

$1 + (0.9)^3 0.1666 = 1.1215$

X, (1)

$1 + (1)^3 0.1666 = 1.1666$

Approximation of Adomian Decomposition Method.

X,(0)

$1 + (0)^2 0.1666 = 1$

X,(0.1)

$1 + (0.1)^2 0.1666 = 1.0017$

X,(0.2)

$1 + (0.2)^2 0.1666 = 1.0067$

X,(0.3)

$1 + (0.3)^2 0.166 = 1.0150$

X,(0.4)

$1 + (0.4)^2 0.1666 = 1.0267$

X,(0.5)

$1 + (0.5)^2 0.1666 = 1.0417$

X,(0.6)

$1 + (0.6)^2 0.1666 = 1.0600$

X,(0.7)

$1 + (0.7)^2 0.1666 = 1.0816$

X, (0.8)

$1 + (0.8)^2 0.1666 = 1.1066$

X, (0.9)

$1 + (0.9)^2 0.1666 = 1.1349$

X, (1)

$1 + (1)^2 0.1666 = 1.6666$

Approximation of the exact solution

X,0

1+0.1666sin0=1

X, 0.1

1+0.1666sin0.2=1.0331

X, 0.2

1+0.1666sin0.4=1.1501

X, 0.3

1+0.1666sin0.6=1.0941

X, 0.4

1+0.1666sin0.8=1.1125

X, 0.5

1+0.1666sin1=1.1402

X, 0,6

1+0.1666sin1.2=1.1553

X, 0.7

1+0.1666sin1.4=1.1642

X, 0.8

1+0.1666sin1.6=1.1666

X, 0.8

1+0.1666sin1.8=1.1673

X,1

1+0.1666sin2=1.1728

Table 4.1

Numerical results for example 4.1

(x)	HAA	ADM	EXACT	HAA ERROR	ADM ERROR
0	1	1	1	0.0E	0.0E
0.1	1.0002	1.0017	1.0331	0.0311	0.3140
0.2	1.0013	1.0067	1.1501	0.1488	0.1434
0.3	1.0045	1.0150	1.0941	0.1018	0.0791
0.4	1.0107	1.0267	1.1125	0.0998	0.0858
0.5	1.0205	1.0417	1.1402	0.1197	0.9850
0.6	1.0360	1.0600	1.1553	0.1193	0.0953
0.7	1.0571	1.0816	1.1642	0.1071	0.0826
0.8	1.0853	1.1066	1.1666	0.0813	0.0600
0.9	1.1213	1.1350	1.1673	0.0460	0.0323
1	1.1666	1.1666	1.1728	0.0062	0.0062

The HAA compares favourably with the ADM and exact solution.

2. Consider the non-linear oscillatory initial value problem

$$y^4 = yy'' + y^2$$

Subject to the initial conditions

$$y(0) = 0. y_1(0) = 1, y_2(0), y_3(0) = 1$$

Source: Liao (2012)

The exact solution is $e^x - 1$, According to the Homotopy Analysis Algorithm, the initial approximation is $y_0(x) = x + \frac{1}{2}x^2 + \frac{1}{6}x^3$

The zero order deformation equation with initial conditions

$$Rm_{(ym-1)} = y'''_{m-1} + \sum_{m=0}^{n-1} y'(x) y''_{m-1}(0) - yj(x)_{m-1} \sum_{j=0}^{m-1} y' - j(x)$$

The solution of the mth order deformation equation for m≥1 is

$$y_m(x) = y_m y_{m-1}(x) + hL - 1R_m y_{m-1} y(x) = -\frac{1}{24hx^4} - \frac{1}{120}hx^5 - \frac{1}{270}hx^6 - \frac{1}{2520}hx^7 + \frac{1}{2016}hx^8 + \ldots$$

The series solution expression can be written in the form $y(x) = y(x) + y'(x) + y''(x) + \ldots..$ and so forth, $y(x) = 1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{750}x^6 + \frac{1}{5040}x^6 + \frac{1}{40320}x^8 + \frac{1}{362880}x^9$

This converges to the Adomian Decomposition Method solution

Approximation of Homotopy Analysis Algorithm

X,(0)

$0 + (0)^2 = 0$

X (0.1)

$0.1 + 0.5(0.1)^2 = 0.1050$

X (0.2)

$0.2 + 0.5(0.2)^2 = 0.2100$

X ,(0.3)

$0.3 + 0.5(0.3)^2 = 0.3450$

X,(0.4)

$0.4 + 0.5(0.4)^2 = 0.4850$

X,(0.5)

$0.5 + 0.5(0.5)^2 = 0.6250$

X,(0.6)

$0.6 + 0.5(0.6)^2 = 0.7800$

X,(0.7)

$0.7 + 0.5(0.7)^2 = 0.9450$

X,(0. 8)

$0.8 + 0.5(0.8)^2 = 1.1200$

X,(0.9)

$0.9 + 0.5(0.9)^2 = 1.3050$

X, 1

$1 + 0.5(1)^2$=1..5000

Approximation of Adomian Decomposition Method

X,(0)

$0 + 0.5(0)^3 = 0$

X,(0.1)

$0.1 + 0.5(0.1^3) = 0.1005$

$0.2 + 0.5(0.2)^3 = 0.2040$

X,(0.3)

$0.3 + 0.5(0.3)^3 = 0.3135$

X,(0.4)

$0.4 + 0.5(0.4)^3 = 0.4320$

X,(0.5)

$0.5 + 0.5(0.5)^3 = 0.5625$

X,(0.6)

$0.6 + 0.5(0.6)^3 = 0.7080$

X,(0.7)

$0.7 + 0.5(0.7)^3 = 0.8715$

X,(0.8)

$0.8 + 0.5(0.8)^3 = 1.0560$

X,(0.9)

$0.9 + 0.5(0.9)^3 = 1.2645$

X,(1)

1+0.5(1)=1.5000

Approximation of the exact solution

$e^{0.1}-1=0.1051$

$e^{0.2}-1=0.2214$

$e^{0.3}-1=0.3499$

$e^{0.4}-1=0.4918$

$e^{0.5}-1=0.6487$

$e^{0.6}-1=0.8221$

$e^{0.7}-1=1.0137$

$e^{0.8}-1=1.2256$

$e^{0.9}-1=1.4596$

$e^{1}-1=1.7183$

Table (4.2)

Numerical results for example 4.2

X	HAA	ADM	EXACT	HAA ERROR	ADM ERROR
0	0.0000	0.0000	0	0.0000	0.0000
0.1	0.0050	0.1005	0.1051	0.1001	0.0046
0.2	0.2100	0.2040	0.2214	0.0114	0.0174
0.3	0.3450	0.3135	0.3499	0.0049	0.0364
0.4	0.4850	0.4320	0.4918	0.0068	0.0598
0.5	0.6250	0.5625	0.6487	0.0237	0.0862
0.6	0.7800	0.7080	0.8221	0.0421	0.1141
0.7	0.9450	0.8715	1.0137	0.0687	0.1422
0.8	1.1200	1.0560	1.2256	0.1056	0.1696
0.9	1.3050	1.2645	1.4596	0.1546	0.1951
1	1.5000	1.5000	1.7183	0.2183	0.2183

CONCLUSION

The proposed algorithm HAA have been successfully applied for the approximation of oscillatory initial value problems. The result obtained is compared with the Adomian Decomposition Method and the exact solution, it was observed that all the problems considered shows that the HAA results compared favourably with the ADM and exact solutions, It is clearly seen that the Homotopy Analysis Algorithm is a cogent and effective algorithm for approximating the numerical (analytic) solution of oscillatory initial value problems, also It could be observed that HAA converges faster and was implemented without any need for discretization of the problem, Therefore for easy of solution to oscillatory IVPs, without tedious algebraic computations, this study recommends HAA for approximating oscillatory IVPs.

REFERENCES

1. Abbasbandy, S. (2006). The application of homotopy analysis method to nonlinear equations arising in heat transfer. Physics Letters A, 360(1):109
2. Abbassbandy, S. (2017). The homotopy analysis method for solving non linear problems. Modelling and analysis of modern fluid problems, 7(2)
3. Bouctayeh A. Twizell E.H (2002) numerical method for solution of special sixth-order boundary value problems. Int compute mail No 43 (207-238) Birkhoff, Khams Malian University. Sabo Adamawa State University, Nigeria in their Article Vol 10, 9734 (AJOAS/2020)
4. Bataineh, A. S., Noorani, M. S. M., and Hashim, I. (2009). Direct solution of nth order ivps by homotopy analysis method. Differential Equations and Nonlinear Mechanics, 2009:1–15.
5. Chioma, I., Ugonna, E., Michael, U., Andrew, O., Ifeyinwa, M., and Ijeoma, U. (2019). Application of homotopy analysis method for solving an seirs epidemic model. Mathematical Modelling and Applications, 4(3):36–48.
6. Fadugba, S. E. and Edeki, S. O. (2022). Homotopy analysis method for fractional barrier option pde. Journal of Physics: Conference Series, 2199(1):012008.
7. Fallahzadeh, A. and Shakibi, K. (2015). A method to solve convection-diffusion equation based on homotopy analysis method. Journal of Interpolation and Approximation in Scientific Computing, 2015(1):1–8.
8. Ghanbari, B. (2014). The convergence study of the homotopy analysis method for solving nonlinear volterra-fredholm integro differential equations. The Scientific World Journal, 2014:1–7.
9. Ghoreishi, M., Ismail, A., and Alomari, A. (2011). Application of the homotopy analysis method for solving a model for hiv infection of cd4+ t-cells. Mathematical and Computer Modelling, 54(11-12):3007–3015.
10. Griffiths D.F, Highman D.J (2010) Numerical methods for ordinary differential equation initial value problems, Springer science and business media
11. HE J.H (2003), Homotopy perturbation method, a new nonlinear analytical technique- Applied mathematics and computation No. 125 (73-79)
12. Imoni S.O, F.O Otunla and Ramamochan (2006) embedded implicit Runge-kutta algorithmic method for solving second-order differential equations – Int.] or compute mathematics vol 8.3 No. 11 (777 – 784)

13. Imoni S.O, Ikhile M.N.O (2014) Zero dissipated RIKKN point of order 5 for solving special second order IVPS Act a uni-Dalakin-Volume: fab. Rev-nat mathematics 53. No. 2 (53-69)
14. Liao S.J (2004) Homotopy Analysis method for non-linear differential equations Springer New York.
15. Liao S.J (2002) Homotopy Analysis Method for nonlinear problems Appl. Math Comput 147.4995
16. Liao, S. (1992). The proposed homotopy analysis technique for the solution of non-linear problems. PHD dissertation. Liao, S. (1997). Homotopy analysis method: A new analytical technique for nonlinear problems. Direct Science, 2(2):95–100.
17. Liao, S. (1999). An explicit, totally analytic approximation of blasius viscous flow problems. Int. J. Nonlin. Mech., 34:759 –778.
18. Liao, S. (2010). An optimal homotopy-analysis approach for strongly nonlinear differential equations. Commun. Nonlinear Sci. Numer. Simulat, 15:2003–2016.
19. Liao, S. (2012). The homotopy analysis method in nonlinear differential equations. Higher Education Press and Springer, Beijing and Heidelberg.
20. Maitama, S. and Zhao, W. (2019). New homotopy analysis transforms method for solving multidimensional fractional diffusion equations. Arab Journal of Basic and Applied Sciences, 27(1):27–44.
21. Marinca, V. and Herisanu, N. (2008). Application of optimal homotopy asymptotic method for solving nonlinear equations arising in heat transfer. Int. Commun. Heat Mass., 35:710 –715.
22. Mkharrib, H. and Salem, T. (2021). New algorithm of the optimal homotopy asymptotic method for solving lane-Emden equations. Journal of Applied Mathematics and Computation, 5(4):237–246. 6
23. Mohyud-Din, S., Hussain, A., and Yildirim, A. (2010). Homotopy analysis method for parametric differential equations. World Applied Sciences Journa, 11(7):851–856.
24. Motsa, S., Sibanda, P., and Shateyi, S. (2010). A new spectral homotopy analysis method for solving a nonlinear second order bvp. Commun. Nonlinear Sci. Numer. Simulat, 15:2293–2302.
25. Nandeppanavar, M. (2016). Applications of homotopy analysis method in science and engineering research problems. Journal of Information Engineering and Applications, (4):21–26. Okposo, N. and Jonathan, A. (2020).
26. WAZWAZ A. W (2007) A not on using Adomian decomposition method for solving oscillatory base temperatures in convective longitudinal fine-Energy conversion and management No. 49, 2910-2916