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Assessing Inflation Rate Forecast Performance: A Setar Model Approach for Sustainable Economic Growth

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Abstract: The relationship between inflation rates and the sustainability of a country's GDP is a crucial aspect of economic development, with profound implications for sustainable growth and development. This research investigates the complex relationship that exists between inflation rates and the sustainable growth of Nigeria's GDP. It also evaluates the prediction accuracy of the Self-Exciting Threshold Autoregressive (SETAR) model. Using Central Bank of Nigeria data from January 2013 to March 2021, a two-regime nonlinear SETAR model is used to capture the nonlinear dynamics that are present in inflation. The study concludes that the SETAR (2;14,13) model is a better predictor of inflation rates after thorough evaluation using metrics like Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE). The study has substantial implications for sustainable development because it offers analysts, investors, and policymakers useful information about how inflation may affect GDP sustainability. This study provides stakeholders with the necessary tools to formulate effective policies aimed at mitigating the negative effects of inflation on sustainable economic growth (SDG 8), thereby fostering long-term development and prosperity. Specifically, the SETAR (2;14,13) model is one of the accurate forecasting models that this study offers.

Keywords: Inflation rate, SETAR model, Forecast comparison, MAE, RMSE, Economic growth

INTRODUCTION

The pursuit of sustainable development is increasingly becoming a focal point in economic research, with a growing recognition of the complex relationships between economic variables and environmental factors. Inflation is the persistent increase in the average price of goods and services within an economy, and it is a crucial macroeconomic factor that has significant effects. Understanding and accurately predicting inflation rates are crucial for economists, decision-

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makers, financiers, and administrative bodies. Accurate models and tools for predicting inflation rates are particularly important in the case of Nigeria, a nation with a dynamic and evolving economy.

To model Nigeria's inflation rate, this study will investigate a sophisticated method. This study explores the realm of Self-Exciting Threshold Autoregressive (SETAR) modeling, a potent method that can represent the regime-switching and nonlinear behaviors frequently displayed by economic variables such as inflation, instead of depending on traditional linear models. This research aims to contribute valuable insights that align with the broader goals of sustainable economic growth and the Earth-friendly agenda by specifically analyzing the environmental influence on inflation rates in Nigeria.

Well into the 1980s, simple linear models continued to dominate macroeconomic forecasting, providing the foundation for forecasting a variety of economic variables (Pettenuzzo & Timmermann, 2017). However, a turning point came when a group of time series analysts were challenged at the 1977 Royal Statistical Society conference. As a result of this challenge, a class of nonlinear models emerged, spurring the development of the threshold concept (Tong, 1978; Tong & Lim, 1980; Ahmed, 2018).

Citing notable references (Ahmed, 2018; Tsay and Tiao, 1984; Campbell & Shiller 2011; Clements, Franses, & Swanson, 2004; Aidoo, 2011; Adedotun, Taiwo, & Olatayo, 2020) underscores the increasingly evident nonlinear nature of most macroeconomic variables. Therefore, nonlinear models have been advocated as better-suited tools for forecasting these variables. However, some dissenting scholars, including (Seetharam, 2016), argue that the superiority of linear or nonlinear models remains inconclusive in terms of forecasting performance.

Due to their inherent irregularity, the asymmetric behavior of time series derived from macroeconomic variables must be studied, modeled, and detected. This irregularity makes simple linear models insufficient for modeling and forecasting purposes.

Artificial Neural Networks (ANN) and Threshold Autoregressive (TAR) models are two examples of nonlinear models that have gained popularity recently and are widely used for forecasting and modeling non-linear time series data. These models are classified as regime-switching models, with additional subcategories such as "Threshold models" and "Markov Switching models," depending on how the state processes in the two models evolve (Onasanya, Olusegun, Adedotun, & Odekina, 2021).

Threshold models are intended to capture irregularities within the time series data generation process. They were first introduced by Tong (1978) and subsequently reviewed by (Potter, 1999). The Self-Exciting Threshold Autoregressive (SETAR) model, which is a special case of the TAR model, is ideal for modeling various macroeconomic variables in Nigeria because it can adjust to structural changes in regimes or discontinuities in data generation processes. However, nonlinear models such as the ANN model can be challenging to comprehend from an economic perspective (Jeong, Siegel, Chen, & Newey, 2020).

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Ahmed (2018) claims that SETAR models perform better in-sample forecasting than their linear counterparts, but their out-of-sample forecasting performance is still unknown (Swanson &White, 1995). This study aims to eliminate uncertainty by thoroughly assessing the in-sample and out-of-sample performance of SETAR models.

This study's main objective is to determine which macroeconomic variables meet the requirements for applying the SETAR model and to evaluate their forecast performance both within and outside the sample. This investigation has the potential to provide insightful guidance for researchers, shareholders, analysts, and administrators navigating the challenging field of macroeconomic forecasting.

This research aims to provide a new perspective on inflation rate modeling in Nigeria by using a SETAR model approach that considers the inherent complexities and irregularities frequently seen in economic data. The results of this study could fundamentally change how inflation rates are perceived and predicted in Nigeria by providing crucial new information for financial analysis and well-informed decision-making.

LITERATURE REVIEW

Imam, Abubakar, & Saleh, (2019) conducted a study on specifying a suitable order and threshold regime number for nonlinear time series models. This study considers the Self-Exciting Threshold Autoregressive (SETAR) model as the nonlinear model. The model is used to fit and forecast simulated nonlinear autoregressive functions at different sample sizes and steps ahead. The SETAR of 2 and 3 autoregressive orders (p) within a regime and 2 and 3 regime orders (d) are fitted at different sample sizes. The relative performances of the SETAR(p, d) are examined to identify the best autoregressive and regime orders within the context of stationarity. The results showed that SETAR(3, 2) and SETAR (2, 2) are the best for fitting small and moderate, and large sample sizes, respectively, in both simulated and real-life data. Moreover, the best forecast models are SETAR (3, 2) followed by SETAR (2, 2) at different steps ahead. Finally, it is revealed that the fitting and forecasting performance of all models increases when sample sizes and the number of steps ahead are increased.

A comprehensive study on the comparative analysis of time series forecasting models was conducted by (Fraz, Iqbal, & Uddin, 2019). In particular, the effectiveness of two different non-linear regime-switching models - the Self-Exciting Threshold Autoregressive (SETAR) and the Markov Regime Switching Autoregressive (MSAR) - with a linear autoregressive (AR) model. This analysis examined a number of crucial macroeconomic factors drawn from both developed and developing nations, including the G7, such as interest rates, GDP growth, industrial production, consumer price index (CPI) inflation, and exchange rates. The dataset consisted of quarterly time series data covering the years 1970-2016. Empirical findings showed that the non-linear SETAR model's forecast performance considerably surpassed that of the non-linear MSAR model and the linear Autoregressive model. The results were evaluated by applying standard

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criteria for forecast accuracy, namely the Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE).

Akintunde, Kgosi, Agunloye, & Olalude, (2019), investigated the analysis of structural changes within Nigeria's Gross Domestic Product data spanning from 1980 to 2017. They precisely evaluated the out-of-sample forecasting performance of a non-linear time series SETAR model, ensuring that all essential theoretical frameworks were explicitly outlined, and stationarity tests were conducted before selecting the model. Their investigation involved a detailed comparison of out-of-sample forecast performances between the conventional linear ARIMA model and the non-linear SETAR model. The empirical demonstration conclusively shows that the non-linear SETAR model outperforms the linear ARIMA model in forecasting Nigeria's Gross Domestic Product.

Umer, Sevil, & Sevil, (2018) compared the effectiveness of linear autoregressive (AR) and smooth transition autoregressive (STAR) models using monthly returns of the FTSE travel, Turkey, and leisure index from April 1997 to August 2016, with the MSCI world index as a proxy for the overall market. The results indicate that the nonlinear LSTAR model cannot improve the linear AR model's out-of-sample forecast, suggesting that there is little benefit to using the LSTAR model in predicting the travel and leisure stock index.

To compare different univariate time series models for future unemployment rate projections, (Davidescu, Apostu, & Paul, 2021) conducted a study. They used out-of-sample data from 2018 to 2020, as well as data from January 2000 to December 2017. The forecasted unemployment rate is for 2021-2022. In in-sample forecast evaluations, the Holt-Winters model performed better than other models. While the SARIMA model offered higher forecast precision, the NNAR model performed better when forecasting outside the sample. The Diebold-Mariano test revealed that SARIMA and NNAR performed differently in forecasting, with NNAR demonstrating the highest level of efficacy in modeling and predicting unemployment rates.

Clements & Smith (2016), examined various methods for obtaining multi-step forecasts ahead for SETAR models. The study was based on existing literature on h-step ahead forecast values. They used various simulation techniques to compare the forecast performance of AR models and SETAR models for the US Gross National Product (GNP). Their findings raised concerns about the resilience of SETAR models in two sample periods, but they also showed that SETAR models could produce highly accurate forecasts. Therefore, they concluded that historical events significantly impacted the accuracy of linear AR model forecasts. They also demonstrated that the out-of-sample forecast performance depends on the type of nonlinearity present in the time series.

According to Gibson & Nur (2011), the in-sample forecast of the STAR model is problematic because it does not replicate the observed behavior of the time series. Karlsson & Karlsson (2016) [23] examined the performance of the VAR model in out-of-sample forecasts for Sweden's unemployment rate, comparing it to the SARIMA and SETAR models. They found that the latter model outperformed the former when using quarterly unemployment data from 1983 to 2010. Additionally, it was found that short-term forecasts perform better than long-term ones.

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Hsu, Li, Lin, Hong, & Huang, (2010) discovered that the SETAR model performed better when examining the out-of-sample forecast of the non-linear time series, while analyzing structural changes in the Taiwan stock market. They compared the out-of-sample forecast of the nonlinear SETAR model and the standard linear ARIMA model using monthly data from January 2005 to December 2009, and used a unit root test to determine which model performed best. A graphical representation of their findings showed that a structural change in the time series occurred in June 2008. Consequently, they created a two-regime SETAR model. In summary, the nonlinear SETAR model outperformed the linear ARIMA model in forecasting the Taiwanese stock market.

Zak (2017), examined how well nonlinear models predicted the exchange rate of the Czech Republic against the euro. For this study, secondary data was collected between 1999 and 2016 from three sources: press releases from the Governing Council of the European Central Bank (ECB), the Statistical Data Warehouse on the ECB website, and the minutes of the board of directors of the Czech National Bank (CNB). The study demonstrated that the simple random walk model outperforms the complex SETAR model, which uses SETAR models. This supports the claim that exchange rates are typically unpredictable.

Firat (2017), used the SETAR modeling process to explain the non-linear pattern in the currency models for the USD/TRY, EUR/USD, and EUR/TRY parities. The study examined how well both linear and nonlinear models predicted the observed values. The non-linear SETAR model outperformed the other nonlinear and linear models. The SETAR model approach to modeling inflation is also linked to the following (Areo & Obindah, 2020; Osabuohien, Obiekwe, Urhie, & Osabohien, 2018; Busayo, Dominic, Olaronke, Oluwatomisin, Bowale & Azuh, 2021).

METHODOLOGY

The study obtained secondary datasets of inflation rates spanning from January 2013 to March 2021 from the website of the Central Bank of Nigeria.

The study analyzed the dynamics of inflation rates using a statistical modeling technique called Self-Exciting Threshold Autoregressive (SETAR) modeling. In this study, the research specifically used a "two-regime" configuration of the SETAR model, which captures nonlinearity in time series data. The SETAR model, which is a specific case of the larger TAR model, extends the Threshold Autoregressive (TAR) model. Tong first proposed the original framework in a landmark 1977 paper, which was expanded upon by (Tong & Lim, 1980). This model provides a better understanding of the time series data generation process, particularly in situations where behavior transitions between distinct regimes occur. SETAR models are widely used to forecast and better understand dynamic regime-switching patterns in higher-order parameters. The "Self-Exciting" part of the name comes from the fact that regime changes in the model are contingent upon the historical values of the time series. The standard representation is (k, p), where k denotes the different regime changes, and p denotes the order of the AR component. This model can be understood as a collection of discrete AR models, each of which is affected by unique threshold

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variables or lagged values established by a delay parameter d. According to Boero & Marrocu (2003), the expression for the two-regime SETAR model with order p is as follows:

$$y_{t} = \begin{cases} \phi_{0}^{(1)} + \sum_{i=1}^{p(1)} \phi_{1}^{(i)} y_{t-i} + \varepsilon_{t}^{(1)} & \text{if } y_{t-d} \leq \tau \\ \\ \phi_{0}^{(2)} + \sum_{i=1}^{p(2)} \phi_{1}^{(i)} y_{t-i} + \varepsilon_{t}^{(2)} & \text{if } y_{t-d} > \tau \end{cases}$$

$$(1)$$

where $\phi_i^{(1)}$ and $\phi_i^{(2)}$ are coefficients in lower and higher regimes

 τ = the threshold values

d = delay parameter

ſ

 y_{t-d} = threshold variable that characterizes the transition between two regimes, $p^{(1)}$ and $p^{(2)}$ are orders of AR in lower and higher regimes respectively and

 ε_t = random error terms which are independently identically distributed. Capturing the irregular behavior of a time series that the "naïve" linear models are unable to capture is one of the main advantages of the nonlinear SETAR model.

3.1 Smooth transition regression models

Van Dijk, Terasvirta & Franses (2002) introduced Smooth Transition Regression models, are a family of nonlinear models that include both the higher regime switching within the time series data and the deterministic shifts in parameters over time. The general STR model for a time

$$X_{t}, t = 1, 2, 3...$$
$$X_{t} = \left(\alpha_{0} + \sum_{i=1}^{p} \alpha_{i} X_{t-i}\right) + \left(\beta_{0} + \sum_{i=1}^{p} \beta_{i} X_{t-i}\right) G\left(Y_{t-d}, \gamma, C\right) + \varepsilon_{i}$$
(2)

Where $G(Y_{t-d}, \gamma, C)$ is the transition with Y_{t-d} as the transition variable which controls the switching point, d is the decay parameter, γ is the smoothing parameter that determines the smoothness of the transition variable, c is the threshold parameter, $\alpha_0, \alpha_1, \dots, \alpha_k$ and $\beta_0, \beta_1, \dots, \beta_p$ are the parameters of the two autoregressive components of the model with optimal lag length p, and ε_i is an error term. The logistic smooth and exponential functions are the two most widely used transition functions.

Logistic Function

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$$G(Y_{t-d},\gamma,C) = \frac{1}{1 + exp\{-(Y_{t-d},-C)\}}, \gamma > 0$$
(3)

Exponential Function

$$G(Y_{t-d},\gamma,C) = \frac{1}{1 + exp\left\{-(Y_{t-d},-C)^{2}\right\}}, \gamma > 0$$
(4)

3.2 ARIMA models

$$X_{t} = \phi_{0} + \sum_{i=1}^{p} \phi_{i} X_{t-i} + \varepsilon_{i} + \sum_{j=1}^{q} \theta_{i} X_{t-i}$$
(5)

where q denotes the Moving Average (MA) model's order and p denotes the Autoregressive (AR) model's order. An expansion of ARIMA(p,q) intended to manage non-stationary time series is the ARIMA(p,d,q) model. Differentiating is frequently used to address non-stationarity, and the degree of integration is indicated by the parameter d in ARIMA(p,d,q). A model for non-stationary data with persistent memory is called ARIMA.

3.3 Order determination

The model order is determined using the Akaike Information Criterion (AIC) (Akaike, 1974), a widely recognized measure for model selection. Additionally, the Final Prediction Error (FPE), introduced by Parzen (1974), offers a comparable approach for evaluating prediction accuracy. The FPE criterion for the kth autoregressive model is provided by

$$FPE_{(k)} = \sigma_k^2 \left[1 - k / N \right] \tag{6}$$

Where σ_k^2 is the unbiased estimator of σ^2 using the kth order model, that is

$$\sigma_k^{\ k} = \frac{RSS_k}{N - K} \tag{7}$$

Similarly, for a *P*th order model,

$$AIC_{(p)} = Nln\sigma_p^2 + 2P \tag{8}$$

$$\sigma_p^2 = \frac{1}{N} \sum_{i=1}^{N} \left((x_i - \sum_{j=1}^{p} \phi_i X_{t-i}) \right)^2$$
(9)

The most precise model for the dataset can be determined by evaluating statistical criteria, with the model possessing the lowest Final Prediction Error (FPE) or Akaike Information Criterion (AIC) often considered as the optimal choice. AIC, a widely recognized measure of model

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performance, has shown a consistent advantage over FPE in numerous empirical studies and is generally preferred as it offers a more robust and reliable method for selecting the most appropriate model for a given dataset.

3.4 Test for linearity

A zero-mean stationary stochastic process (X_t) is considered to be generated by an autoregressive model of order k, often represented as A(k), when it complies with a specific difference equation that characterizes the relationship between its current and past values. This modeling approach, known as autoregressive modeling, is a fundamental concept in time series analysis, and it seeks to capture the dependencies and patterns within the data by expressing them in terms of lagged values up to order k.

$$X_{t} = \phi_{1}X_{t-1} + \phi_{2}X_{t-2} + \ldots + \phi_{p}X_{t-p} + e_{t}$$
(10)

Where e_t is a white noise process with variance σ^2 . Here, e_t will be assumed to be a Gaussian process.

Suppose, in a multiple linear regression, the response variable is given by Y and there is a set of explanatory variables, say $\{X_1, X_2, ..., X_k\}$. The full linear regression model is, given a set of derivations on $\{Y, X_1, X_2, ..., X_k\}$.

$$Y_i = a_1 X_{1i} + a_2 X_{2i} + \dots + a_k X_{ki} + e_i, i = 1, 2, \dots N$$
(11)

Where $e_i \square N(0, \sigma^2)$. The challenge lies in identifying the subset of independent variables that most effectively accounts for the variability in *Y*.

3.5 Forecast Equations used

$$\hat{y}_{i+\tau}(t) = E\left(y_{i+\tau} \setminus y_1, y_2, \dots y_t\right)$$
(12)

Since ARMA models build upon the series a_t , the properties of a_t needs to be revisited. In particular, $a_1, a_2, \dots a_3$... are independent and that future values of a's are independent of the present and the past values of y's, i.e., a_{i+1} is independent of y_t, y_{t-1}, \dots

One-step forecast:

First, we have $y_{t+1} = y_t + a_{t+1} - \theta_1 a_t$

$$\hat{y}_{i+\tau}(t) = E(y_{i+\tau} \setminus y_1, y_2, \dots, y_t) = E(y_t + a_{t+1} - \theta_1 a_t \setminus y_1, y_2, \dots, y_t) = y_t + 0 - \hat{\theta}_1 \hat{a}_t = y_t - \hat{\theta}_1 \hat{a}_t$$
(13)

Two-step forecast

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$$y_{t+2} = y_{t+1} + a_{t+2} - \theta_1 a_t == \hat{y}_{t+2} = \hat{y}_{t+1}(t) + E(a_{t+2}) - \hat{\theta}_1 E(\hat{a}_{t+1}) = \hat{y}_{t+1}(t)$$
(14)

A 'two-regime' SETAR model divides the time series data into two states or regimes. There is a distinct linear autoregressive process linked to each regime. A threshold variable controls the transition between these regimes and determines which regime the data points are independent of. We can more accurately model the inflation rates with this approach, particularly in cases where there is nonlinear behavior in the relationship between the variables. The specific SETAR model used is designated as 'SETAR (2;14,13),' where '2' denotes the number of regimes, and '14' and '13' denote the estimated threshold values for regime switching. Compared to other models, this model configuration performed better in forecasting Nigerian inflation rates.

RESULTS

Preliminary Analysis

The monthly inflation rate data for Nigeria from January 2003 to March 2021 were carefully collected from the Central Bank of Nigeria's official website. The 219 observations in these datasets were carefully divided into two subsets: the training dataset and the testing dataset. After a careful and in-depth examination of the time series plots of these datasets, a discernible finding was made: these datasets showed inherent characteristics of seasonality adjustments and a unique, asymmetric nature.

Table 1 shows that the inflation rate ranged from 5.4 at the lowest value to 19.4 at the highest value. The computed sample moments for the inflation data during the studied period show that the data were leptokurtic, indicating a positive skew with respect to their respective mean values. As the data were leptokurtic, it is evident that the inflation values had more prominent peaks near the inflation rate mean value. The Jarque-Bera test was used to determine whether the inflation data were normal. The resulting p-value of 2.2e - 16 was significantly below the 5% significance level, indicating that the inflation data were not normally distributed.

-	
Mean	11.96183
Standard Deviation	3.27403
Standard Error	0.2212385
Minimum	5.4
Maximum	19.4
Skewness	0.1449035
Kurtosis	2.549547

Table 1. Descriptive Statistics of Inflation Rates

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Jarque-Bera (P-value)	2.2e-16

Figure 1 shows a dynamic pattern of both increasing and decreasing trends in the inflation rate over time. These variations in the inflation rate are due to the erratic state of market circumstances. As the figure prominently illustrates, the time series plot exhibits increased volatility during a period around 2004. Monetary factors, including difficulties in agricultural production and the global economic crisis, could be responsible for this increased volatility, according to (Aidoo, 2011; Ocran,2007) [8] and [34]. A recurring trend is indicated by the inflation rate consistently decreasing from 2004 to 2008. The introduction of specific policies, such as tax reductions, can be attributed to the decrease in prices of goods and services. It is noteworthy that inflation rates appeared to be fairly stable between approximately 2013 and 2017. The plot shows a clear cyclical pattern over time, with periodic fluctuations occurring at regular intervals.



Figure 1. Time series plot of inflation

Non-stationary behavior in Nigeria's inflation is confirmed by Figure 2. This is made clear by the series' steady decline across all lag values in the ACF (Auto-Correlation Function) plot. The partial auto-correlation function, or PACF, plot displays a significant peak at lag 1, along with smaller peaks at a few additional lag points in the series.

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Figure 2. ACF and PACF plots of inflation rates.

In order to achieve stationarity in the series, the data were differenced, as Figure demonstrated the series' non-stationarity.

SETAR modelling of inflation rate

Verifying the existence of nonlinearity in the data is crucial in order to apply the SETAR model to the inflation rate time series. The order of the linear AR(p) model was ascertained in order to evaluate nonlinearity. The AR(p) model with the lowest AIC value among the various lag orders was chosen as the basis for the linear AR(p) order. Using this methodology, it was discovered that the linear AR(14) model was the best option because it had the lowest AIC value. Furthermore, the AR(14) model's P-values for the Tsay and Keenan tests of linearity were both less than the 5% significance level. This result implied threshold nonlinearity in Nigeria's inflation rate. In particular, the Tsay F-test favored the alternative hypothesis of threshold-type nonlinearity, whereas the Keenan-1-degree test supported the null hypothesis of linearity. Compared to the linear model, which is more straightforward, regime-switching models are more appropriate for representing these intrinsic discontinuities and transitions in the inflation rate. Table 2 provides an overview of the linearity tests that were performed on the inflation rate.

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Test	Test statistic	P- value	Order	Decision
Keenan-1-degree	4.2233	0.0042	14	Reject Linearity
Tsay	3.0305	1.936e-12	14	No threshold nonlinearity

 Table 2. Linearity test for inflation rate.

Once the inflation time series' nonlinearity was established, the process of determining which SETAR model would work best for the dataset began. This required figuring out the threshold variable y_{t-d} , where "d" stands for the delay parameter, and the lag order, designated as "p" for each regime. An extensive grid search of different model combinations for SETAR was conducted to identify the tentative model or models that would accurately represent the features of the dataset. Selecting lag orders that minimized the values of AIC and MAPE was one of the selection criteria. Tables 3 below show the condensed findings from the competing models used to forecast the two rates. According to the tables, SETAR (2;14,4), SETAR (2;14,5), and SETAR (2;14,13) with the threshold variables y_{t-4} , y_{t-5} , and y_{t-13} respectively were the best models for explaining the nonlinearity in Nigeria's inflation rate. These three models were therefore designated for additional assessment and examination.

Table 3. AIC and MAPE values of tentative SETAR models.

Model	AIC	MAPE
SETAR(2; 14,4)	-1542.412	1.5681
SETAR(2; 14,5)	-1549.032	1.7785
SETAR(2; 14,13)	-1538.502	1.7001

The conditional least squares method, as described in Tables 4, 5, and 6, is used to estimate the parameters of the candidate models with their corresponding threshold values, in compliance with the methodology of (Franses & Dijk, 2000) [35].

The proportions derived from Table 4, which has a threshold value of 0.03944 and a delay parameter of four, show that data from the low regime, not the high regime, is primarily responsible for the structural breaks in inflation.

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	Low Regime				High Regime	
Coefficient	Estimate	Std. Error	t-value	Estimate	Std. Error	t-value
Constant	-0.0055	0.0063	-0.8821	0.0232	0.0171	1.3566
ϕ_1	0.2311	0.0629	3.6766	-0.1208	0.1617	-0.7467
ϕ_2	-0.0584	0.0648	-0.9018	0.7412	0.1738	4.2647
ϕ_3	0.0559	0.0709	0.7890	-0.0279	0.1055	-0.2640
ϕ_4	0.0544	0.0664	0.8197	0.0228	0.1055	0.2157
ϕ_5	0.1563	0.0908	1.7218	-0.0773	0.1097	-0.7050
ϕ_6	-0.0669	0.0789	-0.8479	0.0437	0.0880	0.4965
ϕ_7	0.0549	0.0688	0.7978	-0.0933	0.1081	-0.8625
ϕ_8	0.0568	0.0683	0.8306	0.0364	0.0963	0.3776
ϕ_9	-0.0840	0.0681	-1.2336	0.1066	0.0933	1.1422
ϕ_{10}	0.1396	0.0644	2.1678	-0.0387	0.1044	-0.3704
ϕ_{11}	-0.0563	0.0617	-0.9115	-0.0807	0.1222	-0.6606
ϕ_{12}	-0.3874	0.0628	-6.1712	-0.3996	0.1161	-3.441
ϕ_{13}	0.1143	0.0801	1.4268	0.0248	0.1104	0.2249
ϕ_{14}	-0.0341	0.0666	-0.5115	0.1700	0.1435	1.1851
	Threshold value = 0.03944					
Proportion		75.09%			24.91%	

Table 4. Estimated Parameters of SETAR (2;14,4) for inflation.

Notably, more than half of the data points in Table 5 show discontinuities and switches in the low regime. This observation implies that when using a delay parameter of five and a threshold value of 0.01342, the earlier years of the inflation data exhibit clear discontinuities within the dataset.

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	Low Regime			High Regin	High Regime		
Coefficient	Estimate	Std. Error	t-value	Estimate	Std. Error	t-value	
Constant	-0.0137	0.0078	-1.7433	0.0225	0.0111	2.0377	
ϕ_1	0.3381	0.0789	4.2820	0.1130	0.0856	1.3205	
ϕ_2	-0.0597	0.0686	-0.8706	0.3225	0.1202	2.6834	
ϕ_3	0.0873	0.0737	1.1841	0.0315	0.0864	0.3648	
ϕ_4	0.1221	0.0776	1.5737	-0.0060	0.0772	-0.0775	
ϕ_5	-0.0176	0.0699	-0.2516	0.0979	0.0944	1.0371	
ϕ_6	-0.0927	0.0989	-0.9375	-0.0280	0.0918	-0.3054	
ϕ_7	0.0680	0.0808	0.8408	-0.0519	0.0769	-0.6746	
ϕ_8	0.0155	0.0712	0.2182	0.1419	0.0912	1.5565	
ϕ_9	-0.0293	0.0755	-0.3884	0.0038	0.0779	0.0490	
ϕ_{10}	0.2872	0.0820	3.5007	-0.0726	0.0723	-1.0046	
ϕ_{11}	-0.1464	0.0678	-2.1583	-0.0382	0.0892	-0.4283	
ϕ_{12}	-0.3134	0.0739	-4.2402	-0.5414	0.0792	-6.8377	
ϕ_{13}	0.0797	0.0780	1.0217	0.2289	0.0926	2.4729	
ϕ_{14}	-0.0064	0.0792	-0.0808	0.0311	0.0914	0.3403	
	Threshold value = 0.01342					•	
Proportion	61.77%			38.23%			

	Table 5. Estimated	Parameters of	SETAR(2; 14	5) for Inflation
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In contrast to Tables 4 and 5, Table 6 indicates that discontinuities and switching regimes were observed in over 80% of the inflation data in the higher regimes relative to the low regime.

Additionally, the study examines the models in SETAR modeling to ensure that they follow all of the model assumptions that were previously mentioned. As a result, the model residuals were scrutinized to ensure that there was no autocorrelation, that the mean was zero, and that the residual variance was constant. Furthermore, we conducted ARCH-LM and Ljung-Box tests on the residuals, each according to the previously described. According to the null hypothesis of random and uncorrelated residuals, the results in Table 6 support the lack of autocorrelation and the randomness of residuals in all tentative models. SETAR (2;14,4) clearly failed the ARCH-test for a lag order of 12, indicating the possibility of autocorrelation upon closer examination of the table.

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	Low Regim	e		High Regin	ie	
Coefficient	Estimate	Std. Error	t-value	Estimate	Std. Error	t-value
Constant	-0.0412	0.0326	-1.2643	-0.0032	0.0053	-0.5957
ϕ_1	-0.0105	0.1322	-0.0792	0.3050	0.0673	4.5348
φ ₂	0.1790	0.0937	1.9099	0.0405	0.0825	0.4916
<i>φ</i> ₃	0.2177	0.0862	2.5269	0.0254	0.0740	0.3428
ϕ_4	-0.0780	0.1049	-0.7434	0.1051	0.0715	1.4697
ϕ_5	-0.1625	0.1468	-1.1070	0.0750	0.0635	1.1805
ϕ_6	0.2319	0.1053	2.2024	0.0294	0.0644	0.4567
φ ₇	-0.1882	0.0957	-1.9670	0.0611	0.0687	0.8895
ϕ_8	-0.1880	0.1425	-1.3193	0.0794	0s.0612	1.2972
ϕ_9	-0.0910	0.1105	-0.8232	0.0033	0.0617	0.0532
ϕ_{10}	0.0822	0.0967	0.8496	0.1159	0.0682	1.6990
ϕ_{11}	-0.1485	0.1287	-1.1543	-0.0950	0.0638	-1.4897
ϕ_{12}	0.3674	0.1453	-2.5280	-0.3932	0.0632	-6.2224
φ ₁₃	0.1225	0.1223	1.0013	0.0935	0.0761	1.2296
ϕ_{14}	-0.1817	0.1972	-0.9214	0.0223	0.0804	0.2779
	Threshold value = -0.07092					1
Proportion	17.41%			82.59%		

Table 6. Estimated parameters of *SETAR*(2; 14, 13) for inflation.

According to the relatively low Ljung-Box test statistics for the three SETAR models in Table 7, there is no discernible autocorrelation in the residuals.

Model	Ljung-Box test	ARCH-LM test
SETAR(2; 14,4)	0.8372	0.0031
SETAR(2; 14,5)	.8668	.4481
SETAR(2;14,13)	.9777	.5916

Table 7. Residual diagnostics of tentative SETAR models for inflation.

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The ARCH-LM test indicates that there are differences between the models regarding the presence of conditional heteroskedasticity. A low p-value in the first model (SETAR 2;14,4) indicates the existence of conditional heteroskedasticity. Higher p-values for the second and third models (SETAR 2;14.5 and SETAR 2;14,13) suggest that conditional heteroskedasticity is not as concerning.

Table 8 presents the outcomes of model performance metrics, both in- and out-of-sample, for three SETAR models, each with its own set of parameters. These metrics evaluate the models' goodness of fit and accuracy. The findings indicate that, although all three models have comparable in-sample model fits, SETAR(2;14,4) performs the best in terms of out-of-sample forecasting.

	Out-of-sample		In-sample	
Model	RMSE	MAE	RMSE	MAE
SETAR(2; 14,4)	.0016	.0351	.0059	.0529
SETAR(2; 14,5)	.0022	.0395	.0056	.0515
SETAR(2; 14,13)	.0018	.0354	.0058	.0515

Table 8. Forecast results of competing SETAR models for inflation

The models chosen from the two modeling approaches demonstrated a good fit to the inflation data while meeting all model assumptions. The two models were compared to determine which is more significant, even though the nonlinear SETAR model performed better for inflation rates than the linear SARIMA model by their least error terms. The Diebold-Mariano test was used for this comparison, assuming that there is a significant difference between the two models. Table 9 results show that every P-value across the five forecast horizons was rejected.

Table 9: Forecast accuracy test for inflation
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Forecast Horizon	DM statistic	P-value
1	-0.2489	.8038
2	-0.2398	.8108
3	-0.2444	.8072
4	-0.2768	.7823
5	-0.2959	.7677

DISCUSSION

An initial examination of the inflation rate datasets revealed that their distribution did not align with a normal pattern. This non-normality was further confirmed through the Jarque-Bera test,

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indicating the presence of substantial positive skewness, with values exceeding three. Furthermore, the time series plots for inflation rate displayed nonstationary behavior, including the application of logarithmic transformations and differencing to induce stationarity.

Upon transformation, the differenced series exhibited a distinctive sine-wave-like structure in the Auto-Correlation Functions (ACF), signifying the coexistence of both seasonal and non-seasonal patterns within the series. Notably, the non-seasonal differencing alone sufficed to render the series stationary, obviating the need for further seasonal differencing. These transformed and differenced series were subsequently used to construct suitable models following the established Box-Jenkins methodology. All the selected candidate models for the inflation rate demonstrated favorable performance, as indicated by the ACF plots of residuals, suggesting their potential as robust forecasting models.

To implement the SETAR model, it was essential to verify the nonlinear nature of the inflation rate dataset. This verification was carried out through linearity tests, specifically employing the Keenan and Tsay tests for linearity. The resulting P-values were consistently below 5%, supporting the assertion that the dataset conforms to a threshold nonlinear structure.

The significance of obtaining accurate models for forecasting cannot be overstated, as it holds paramount importance for researchers, policymakers, investors, and governments alike. Inflation rate, being a pivotal driver of an economy, is of particular relevance to Nigeria. The ability to forecast this variable effectively serves as a guiding tool for financial and economic analysts, enabling them to make informed decisions and offer crucial policy recommendations aimed at mitigating inflation rates. In this context, the study has identified the SETAR (2;14,13) model as the most proficient in predicting inflation rates, marking a substantial contribution to the field of economic forecasting.

IMPLICATION TO RESEARCH AND PRACTICE

This study's analysis of Nigeria's inflation data using SETAR models reveals several critical insights with broader implications for future research. First, the research highlights the importance of incorporating nonlinearity and regime-switching behavior in time series models for inflation, especially in contexts where economic shocks, policy changes, and structural breaks are common. As traditional linear models fail to capture such complexities, future research should focus on non-linear models like SETAR to better predict inflation trends in similar economies. Moreover, this study demonstrates the effectiveness of AIC and MAPE criteria for model selection, which provides a methodological basis for scholars working on time series forecasting to improve the accuracy and robustness of their models. Lastly, the results suggest that focusing on low-regime factors, as shown in the estimated parameters of the SETAR models, could be key for understanding inflationary behaviors in volatile markets.

The findings also have practical implications for policymakers and financial analysts. Given that nonlinearity plays a substantial role in the behavior of Nigeria's inflation rate, policymakers should consider economic indicators that exhibit sudden shifts and discontinuities. This means that

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inflation-targeting frameworks and monetary policies need to account for such nonlinear patterns to be more adaptive and responsive to economic transitions. Moreover, the study's findings suggest that in periods of high volatility, especially under conditions like global crises or local production challenges, regulatory bodies could benefit from monitoring inflation using nonlinear models to anticipate economic downturns more accurately. Practitioners in financial planning and forecasting could adopt similar models for inflation rate prediction, leading to more informed decisions on investments, interest rates, and risk management.

CONCLUSION

This study examines the use of nonlinear models in predicting Nigerian inflation rates, focusing on the complexity of macroeconomic variables. The data was collected from the Bank of Nigeria website from January 2003 to March 2021 and transformed using the Box-Jenkins technique. The results show that the data does not fit a normal distribution and has strong positive skewness, indicating non-normality.Time series plots also reveal non-stationary behavior, necessitating the use of differencing and logarithmic transformations. Auto-correlation functions of the differenced series reveal a distinct sine-wave-like pattern indicating both seasonal and non-seasonal components. The Self-Exciting Threshold Autoregressive (SETAR) model is identified as the most effective at predicting inflation rates. The study's findings have implications beyond academia, providing insights for policymakers, investors, and governments to make informed decisions and create policies that effectively address inflation rates.

The SETAR (2;14,13) model is found to be the most effective in forecasting inflation rates, a significant advancement in economic forecasting. The use of sophisticated non-linear models is crucial for accurate forecasting in a constantly changing economic environment, influencing financial and economic choices that determine the country's course. Policymakers, financial analysts, and investors are encouraged to adopt advanced techniques for forecasting inflation rates, leading to more precise forecasts and informed decision-making towards sustainable and eco-friendly economic growth in Nigeria.

FURTHER RESEARCH

This research opens up avenues for further investigation in several areas. Future research could expand on the current work by exploring the applicability of other regime-switching models, such as Markov-Switching ARMA or SETAR-GARCH models, to capture both volatility and nonlinearity in inflation rates. Additionally, it would be insightful to compare the effectiveness of the SETAR model in forecasting inflation across different sectors of the Nigerian economy, such as agriculture, manufacturing, or services, to better understand inflationary pressures in each domain. Researchers could also explore whether similar nonlinearity patterns emerge in inflation rates in other African economies, potentially leading to more regionally tailored economic models for policy analysis. Lastly, incorporating external factors such as global oil prices, exchange rate

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volatility, and fiscal policies could enhance the precision and applicability of inflation forecasting models.

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